Supplementary webappendix
to
"Questioning the regressivity of tobacco taxes: a distributional accounting impact model of increased tobacco taxation" by Stéphane Verguet, Patrick K.A. Kearns, and Vaughan W. Rees

We present in this supplementary webappendix the mathematical derivations constituting the distributional accounting impact model of increased tobacco taxation, along with a few additional results and figures not displayed in the main text of the paper.

1. Mathematical examination of regressivity and net change in cigarette taxes

Examining the regressivity of the net change in tobacco taxes ΔT implies exploring the monotonous character, with respect to income y, of the function $\Delta T/y$, as defined in the main text of the paper (equation 3). That is, regressivity would imply that $\Delta T/y$ would decrease as income y increases. Mathematically, this means that we need to explore the sign of the first derivative with respect to income y of the function $\Delta T/y$:

$$\tau(y) = \frac{\partial}{\partial y}\left(\frac{\Delta T}{y}\right) = \delta t * (-p_1 * S_1(y) - \delta t * \varepsilon(y) * S_1(y) - p_1 * t_1 * \varepsilon(y) * S_1(y) + \delta t * y * S_1(y) * \varepsilon'(y) + p_1 * t_1 * y * S_1(y) * \varepsilon'(y) + p_1 * y * S_1{}'(y) + \delta t * y * \varepsilon(y) * S_1{}'(y) + p_1 * t_1 * y * \varepsilon(y) * S_1{}'(y))/(p_1 * y^2) , \quad \text{(S.1)}$$

where p_1 is the retail price of cigarettes (before increased taxation), t_1 is the tax share within the retail price p_1 of cigarettes (before increased taxation), δt $(= \delta p)$ is the change in the retail price of cigarettes (through taxation), $S_1(y)$ is total cigarette consumption (before increased taxation) which varies with income, $S_1{}'(y)$ is the first derivative of total cigarette consumption which corresponds to an income gradient in cigarette consumption, $\varepsilon(y)$ is the price elasticity of demand for cigarettes which varies with income, and $\varepsilon'(y)$ is the first derivative of the price elasticity which corresponds to an income gradient in price elasticity.

2. Mathematical formulation of net change in expenditures on cigarettes

Consistent with the estimation of total annual taxes on cigarettes, before increased tobacco taxation, the total annual expenditures on cigarettes borne by an individual with income y (denoted $C_1(y)$) corresponds to the total number of cigarettes consumed annually by the individual multiplied by the retail price of cigarettes. In other words, at the population level:

$$C_1(y) = S_1(y) * p_1 \,. \quad \text{(S.2)}$$

After increased tobacco taxation, at the population level, the total annual cigarette expenditures borne by an individual with income y (denoted $C_2(y)$) corresponds to the reduced number of cigarettes consumed annually by the individual ($S_1(y) * (1 + \frac{\delta t}{p_1} * \varepsilon(y))$) multiplied by the new retail price of cigarettes ($p_1 + \delta t$). Therefore, at the population level:

$$C_2(y) = S_1(y) * (1 + \frac{\delta t}{p_1} * \varepsilon(y)) * (p_1 + \delta t) \,, \qquad \text{(S.3)}$$

where p_1 is the retail price of cigarettes (before increased taxation), δt is the change in the retail price (through taxation), $S_1(y)$ is total cigarette consumption which varies with income, and $\varepsilon(y)$ is the price elasticity of demand for cigarettes which varies with income.

Subsequently, at the population level, we can derive the net change in cigarette expenditures borne by an individual with income y (denoted $\Delta C(y)$) in the following way:

$$\Delta C(y) = C_2(y) - C_1(y) = S_1(y) * \frac{\delta t}{p_1} * [p_1 + \varepsilon(y) * (p_1 + \delta t)] \,. \qquad \text{(S.4)}$$

Lastly, examining the regressivity of the net change in cigarette expenditures ΔC implies exploring the monotonous character, with respect to y, of the function $\Delta C/y$. This means that we need to explore the sign of the first derivative with respect to y of $\Delta C/y$:

$$\kappa(y) = \frac{\partial}{\partial y}\left(\frac{\Delta C}{y}\right) = \delta t * (-p_1 * S_1(y) - \delta t * \varepsilon(y) * S_1(y) - p_1 * \varepsilon(y) * S_1(y) + \delta t * y * S_1(y) * \varepsilon'(y) + p_1 * y * S_1(y) * \varepsilon'(y) + p_1 * y * {S_1}'(y) + \delta t * y * \varepsilon(y) * {S_1}'(y) + p_1 * y * \varepsilon(y) * {S_1}'(y))/(p_1 * y^2) \,, \quad \text{(S.5)}$$

where p_1 is the retail price of cigarettes (before increased taxation), δt $(= \delta p)$ is the change in the retail price (through taxation), $S_1(y)$ is total cigarette consumption (before increased taxation) which varies with income, ${S_1}'(y)$ is the first derivative of total cigarette consumption which corresponds to an income gradient in cigarette consumption, $\varepsilon(y)$ is the price elasticity of demand for cigarettes which varies with income, and $\varepsilon'(y)$ is the first derivative of the price elasticity which corresponds to an income gradient in price elasticity.

3. Studying the potential regressivity of increased tobacco taxation

We assume, for simplicity that price elasticity of demand for cigarettes is linearly changing with income y: $\varepsilon(y) = \varepsilon_0 + \varepsilon_g * y$ (ε_g is the income gradient in price elasticity and ε_0 the price

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elasticity for individuals with lower income (i.e. the poorest)); and that smoking and consumption would linearly vary with income: $S_1(y) = s_0 + s_g * y$ (s_g is the income gradient in cigarette consumption and s_0 cigarette consumption for the poorest). We can then derive the following mathematical expressions for $\tau(y) = \frac{\partial}{\partial y}(\frac{\Delta T(y)}{y})$ and $\kappa(y) = \frac{\partial}{\partial y}(\frac{\Delta C(y)}{y})$:

$$\tau(y) = [-s_0 * \left(p_1 + \varepsilon_0 * (\delta t + p_1 * t_1)\right) + \varepsilon_g * s_g * (\delta t + p_1 * t_1) * y^2] * \delta t/(p_1 * y^2), \quad \text{(S.6)}$$

$$\kappa(y) = [-s_0 * \left(p_1 + \varepsilon_0 * (\delta t + p_1)\right) + \varepsilon_g * s_g * (\delta t + p_1) * y^2] * \delta t/(p_1 * y^2) \; . \quad \text{(S.7)}$$

We denote $r = \frac{\delta t}{p_1}$, the relative change in the retail price, and we normalize income: $y \, \epsilon \, [0; 1]$, so that 0 and 1 represent lower (i.e. the poorest) and higher income (i.e. the richest), respectively.

We study two scenarios. In Scenario 1, we assume ${S_1}'(y) = s_g = 0$ or $S_1(y) = s_0$: in other words, total cigarette consumption is constant across incomes. In Scenario 2, we relax this assumption with $s_g \neq 0$, and $S_1(y) = s_0 + s_g * y$. For each scenario, we examine the net change in taxes (via equation S.6) and the net change in expenditures on cigarettes (via equation S.7), respectively.

3.1. Net change in additional taxes

For Scenario 1, the case of constant total cigarette consumption across income ($s_g = 0$), we obtain:

$$\tau(y) = -s_0 * \delta t * (1 + \varepsilon_0 * (r + t_1)]/y^2 \; . \quad \text{(S.8)}$$

Because $s_0 > 0$, $\delta t > 0$, and $y^2 > 0$, the sign of $\tau(y)$ in (S.8) will be determined by the sign of $A(\varepsilon_0; r; t_1) = -[1 + \varepsilon_0 * (r + t_1)]$. $A(\varepsilon_0; r; t) = 0$ if and only if $r = -(1 + \varepsilon_0 * t_1)/\varepsilon_0$: this indicates the "neutrality" frontier (i.e. $\tau(y) = \frac{\partial}{\partial y}\left(\frac{\Delta T(y)}{y}\right) = 0$), which corresponds to the case of tax neutrality using our income-share accounting definition of tax burden.

Therefore, for net tobacco taxes to be progressive ($\tau > 0$), r needs to increase as ε_0 increases, from $r = 1/2$ (50% relative price increase) when $\varepsilon_0 = -2.00$ (price elasticity among the poorest of -2.00), to $r = 1$ when $\varepsilon_0 = -1.00$, and to $r = 2$ when $\varepsilon_0 = -0.50$ (the $(r; \varepsilon_0)$ parameter values computed correspond to $t_1 = 0$; see Figure 2 in the main text). This means that progressivity ($\tau > 0$) is maintained as long as ε_0 and r are sufficiently large (above the beam of curves on Figure 2), and this would be mitigated by the initial level of taxes t_1 within the retail price (Figure 2): a higher level of t_1 would require lower neutrality frontier values for ε_0 and r (Figure S.1).

For Scenario 2, when total cigarette consumption varies across income (or $s_g \neq 0$), we obtain:

$$\tau(y) = \left(-s_0 * \left(1 + \varepsilon_0 * (r + t_1)\right) + \varepsilon_g * s_g * (r + t_1) * y^2\right) * \delta t / y^2 . \quad \text{(S.9)}$$

Therefore, we need to study the sign of the following function:

$$A\left(\varepsilon_0; \varepsilon_g; s_0; s_g; r; t_1; y\right) = -s_0 * \left(1 + \varepsilon_0 * (r + t_1)\right) + \varepsilon_g * s_g * (r + t_1) * y^2 . \quad \text{(S.10)}$$

We first solve $A\left(\varepsilon_0; \varepsilon_g; s_0; s_g; r; t_1; y\right) = 0$ with respect to y (i.e. seeking y for which $\tau(y) = \frac{\partial}{\partial y}\left(\frac{\Delta T(y)}{y}\right) = 0$)), and we obtain the cutoff income y_c:

$$y_c = \sqrt{s_0}\sqrt{\frac{1+\varepsilon_0(r+t_1)}{\varepsilon_g s_g(r+t_1)}}, \qquad \text{(S.11)}$$

which is defined if and only: $1 + \varepsilon_0(r + t_1) < 0$. This necessary condition was examined previously (Scenario 1). When this necessary condition is fulfilled, we see that net tobacco taxes will be progressive for $A(\varepsilon_0; \varepsilon_g; s_0; s_g; r; t_1; y) > 0$ if and only if:

(1) $1 + \varepsilon_0(r + t_1) < 0$ (necessary condition of Scenario 1: ε_0 and r being sufficiently large),

(2) $y < y_c$ (income needs to be inferior to the cutoff income parametrically defined in S.11).

With normalized income $0 \leq y \leq 1$ ($y = 0$ corresponding to the poorest income; $y = 1$ corresponding to the richest income) then $A(\varepsilon_0; \varepsilon_g; s_0; s_g; r; t_1; y) > 0$ as long as $y_c > 1$ (Figure 3 in the main text).

We then see that net taxes will always be progressive as long as ε_g and s_g remain within a certain parameter space (yellow-orange area above the horizontal blue plateau on Figure S.2), after the first necessary condition of ε_0 and r being sufficiently large is fulfilled.

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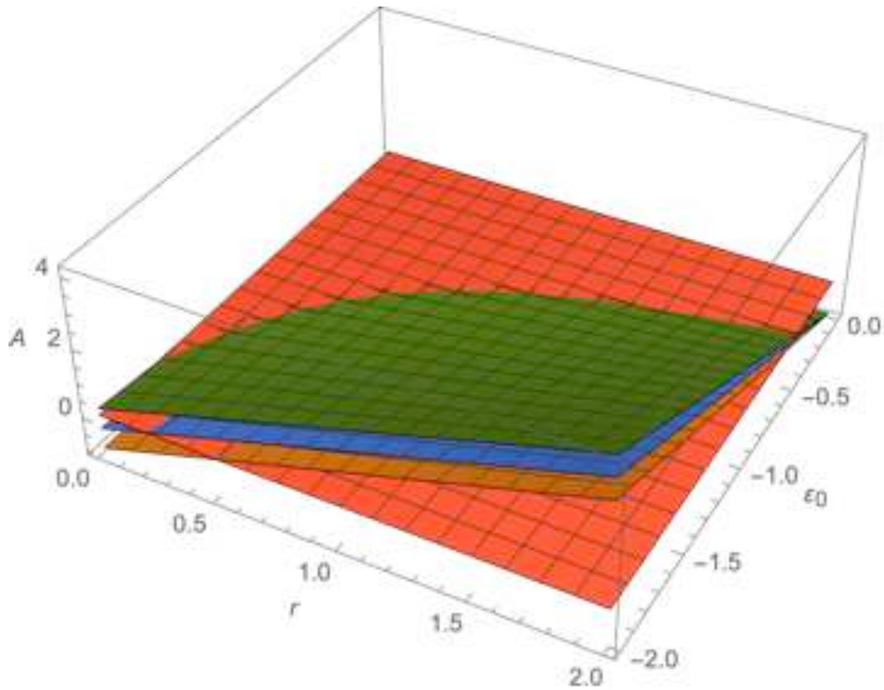
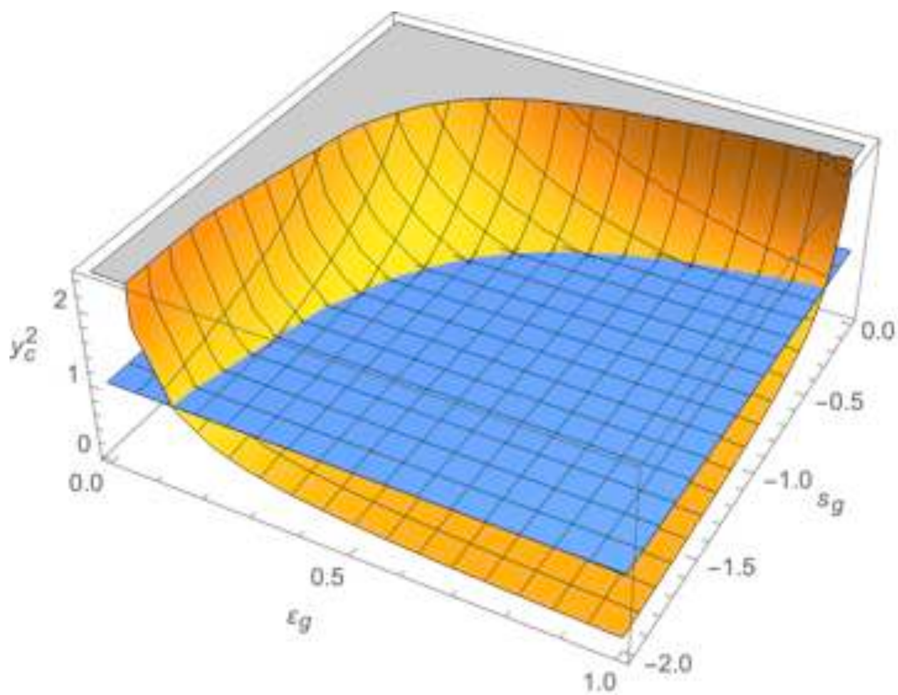
Figure S.1. Value of the function $A(\varepsilon_0; r; t_1) = -[1 + \varepsilon_0(r + t_1)]$ as a function of relative price increase (r, varying from 0 to 2) and price elasticity of demand for the poorest (ε_0, varying from -2.0 to 0.0) for different initial tax shares t_1 (0 in orange, 0.30 in blue, 0.60 in green) within the initial retail price. The red horizontal plateau at $A = 0$ separates the parameter spaces $(\varepsilon_0; r)$ for which $A > 0$ (progressive net taxes) and $A < 0$ (regressive net taxes).



Note: for an initial level of taxes t_1 within the retail price, net tobacco taxes can be progressive if r (relative price increase) increases as ε_0 (price elasticity among the poorest) increases: for example, from r =50% when $\varepsilon_0 = -2.00$, to r =100% when $\varepsilon_0 = -1.00$, and to r =200% when $\varepsilon_0 = -0.50$. This means that progressivity is maintained as long as ε_0 and r are sufficiently large; and a higher initial level of taxes t_1 would require lower neutrality frontier values for ε_0 and r (from orange to blue to green intersecting planes).

Figure S.2. Value of the function ${y_c}^2 = s_0 \frac{1+\varepsilon_0(r+t_1)}{\varepsilon_g s_g(r+t_1)}$ (yellow-orange surface) as a function of the income gradient in price elasticity of demand (ε_g) and the income gradient in total cigarette consumption (s_g) for the following parameter values: tax share $t_1 = 0.50$; price elasticity of demand for the poorest of $\varepsilon_0 = -1.00$; relative price increase $r = 0.60$; and $s_0 = 0.30 * 10$ (smoking prevalence of 30% and daily consumption of 10 cigarettes). The horizontal blue plateau separates the parameter spaces ($\varepsilon_g; s_g$) for which $y_c > 1$ (fully progressive net tax increases) and $y_c < 1$ (partially progressive net tax increases).



Note: after the first necessary condition of ε_0 (price elasticity among the poorest; -1.00) and r (relative price increase; 0.60) being sufficiently large is fulfilled, net tobacco taxes can be fully progressive for a combination of values of ε_g (income gradient in price elasticity) and values of s_g (income gradient in smoking consumption) that yields a value of y_c superior to 1 or above the horizontal blue plateau.

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3.2. Net change in cigarette expenditures

In Scenario 1 – constant total cigarette consumption across income (or $s_g = 0$) – we obtain:

$$\kappa(y) = -s_0 * \delta t * (1 + \varepsilon_0 * (r + 1))/y^2 \ . \qquad \text{(S.12)}$$

Hence, because $s_0 > 0$, $\delta t > 0$, and $y^2 > 0$, the sign of (S.12) will be determined by the sign of $A(\varepsilon_0; r) = -[1 + \varepsilon_0(r + 1)]$. We have $A(\varepsilon_0; r) = 0$ (corresponding to $\kappa(y) = \frac{\partial}{\partial y}\left(\frac{\Delta C(y)}{y}\right) = 0$) when $r = -(1 + \varepsilon_0)/\varepsilon_0$ (Figure 4 in the main text).

As price elasticity for the poorest (ε_0) increases (decreases in absolute value), r needs to increase to maintain $\kappa > 0$ (i.e. net cigarette expenditures to be progressive), for example, from $r = 0$ (0% relative price increase) when $\varepsilon_0 = -1.00$ (elasticity for the poorest of -1.00), to $r = 1$ when $\varepsilon_0 = -0.50$ (elasticity for the poorest of -0.50), to $r = 2$ when $\varepsilon_0 = -0.33$. We see that progressivity in net cigarette expenditures would require large enough ε_0: for example, for a relative price increase of 50% ($r = 0.50$), ε_0 would need to be greater in absolute value than 0.66 ($\varepsilon_0 < -0.66$) so that increased taxation becomes progressive for the net change in cigarette expenditures.

In Scenario 2 – varying total cigarette consumption across income ($s_g \neq 0$) – we obtain:

$$\kappa(y) = \delta t * [-s_0 * \left(1 + \varepsilon_0 * (r + 1)\right) + \varepsilon_g * s_g * r * (r + 1) * y^2]/y^2 \ , \qquad \text{(S.13)}$$

and we need to study the sign of the following function:

$$A\left(\varepsilon_0; \varepsilon_g; s_0; s_g; r; y\right) = -s_0 * (1 + \varepsilon_0 * (r + 1)) + \varepsilon_g * s_g * (r + 1) * y^2 \ . \qquad \text{(S.14)}$$

When solving $A(\varepsilon_0; \varepsilon_g; s_0; s_g; r; y) = 0$ with respect to y (seeking y for which $\kappa(y) = \frac{\partial}{\partial y}\left(\frac{\Delta C(y)}{y}\right) = 0$), we obtain the cutoff income y_c:

$$y_c = \sqrt{s_0}\sqrt{\frac{1+\varepsilon_0(r+1)}{\varepsilon_g s_g (r+1)}}, \qquad \text{(S.15)}$$

which is defined if and only: $1 + \varepsilon_0(r + 1) < 0$. This necessary condition was examined previously (Scenario 1; Figure 4 in the main text). When that condition is fulfilled, we see that net cigarette expenditures will be progressive or $A(\varepsilon_0; \varepsilon_g; s_0; s_g; r; y) > 0$ if and only if:

(1) $1 + \varepsilon_0(r + 1) < 0$ (necessary condition of Scenario 1: ε_0 and r being sufficiently large),

(2) $y < y_c$ (income needs to be inferior to the cutoff income parametrically defined in S.15).

With normalized income $0 \leq y \leq 1$ ($y = 0$ corresponding to the poorest income; $y = 1$ corresponding to the richest income) then $A(\varepsilon_0; \varepsilon_g; s_0; s_g; r; y) > 0$ as long as $y_c > 1$ (Figure 5 in the main text). We then see that net expenditures will be progressive as long as ε_g and s_g remain within a certain parameter space (yellow-orange area above the horizontal blue plateau on Figure S.3), after the first necessary condition of ε_0 and r being sufficiently large is fulfilled.

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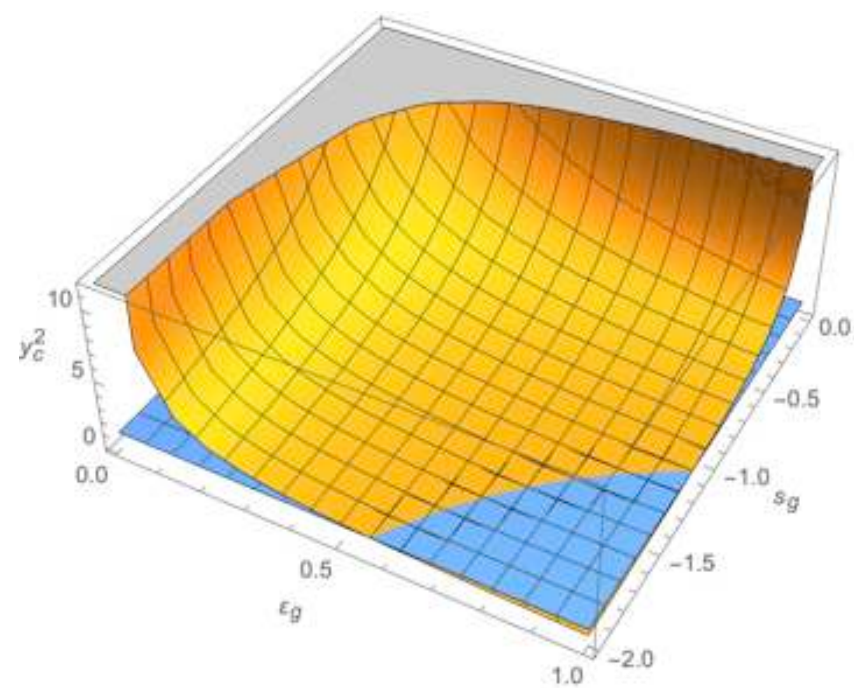
Figure S.3. Value of the function ${y_c}^2 = s_0 \frac{1+\varepsilon_0(r+1)}{\varepsilon_g s_g(r+1)}$ (yellow-orange surface) as a function of income gradient in price elasticity of demand (ε_g) and income gradient in total cigarette consumption (s_g), for the following parameter values: price elasticity of demand for the poorest of $\varepsilon_0 = -1.00$; relative price increase $r = 0.60$; and $s_0 = 0.30 * 10$ (smoking prevalence of 30% and daily consumption of 10 cigarettes). The horizontal blue plateau separates the parameter spaces $(\varepsilon_g; s_g)$ for which $y_c > 1$ (fully progressive net cigarette expenditures) and $y_c < 1$ (partially progressive net cigarette expenditures).



Note: after the first necessary condition of ε_0 (price elasticity among the poorest; -1.00) and r (relative price increase; 0.60) being sufficiently large is fulfilled, net cigarette expenditures can be fully progressive for a combination of values of ε_g (income gradient in price elasticity) and values of s_g (income gradient in smoking consumption) that yields a value of y_c superior to 1 or above the horizontal blue plateau.

4. Examining the likelihood of progressivity in net cigarette expenditures

We report on the likelihood of progressivity in the net change in cigarette expenditures in the case where total cigarette consumption varies across income groups ($s_g \neq 0$; Scenario 2). We take $\varepsilon_0 = -1.00$ (price elasticity of demand among the poorest) and $r = 0.60$ (relative price increase of 60%), so that the first condition of ε_0 and r being sufficiently large is realized (consistent with Scenario 1 ($s_g = 0$); Figure 4, main text). We then examine the likelihood of progressivity in the net change in cigarette expenditures – the ratio of net cigarette expenditures relative to income increases with income – when we vary values of s_g (income gradient in smoking consumption) and values of ε_g (income gradient in price elasticity).

For this purpose, we study the value of the cutoff function ${y_c}^2 = s_0 \frac{1+\varepsilon_0(r+1)}{\varepsilon_g s_g (r+1)}$ as a function of ε_g and s_g for different values of smoking consumption among the poorest individuals s_0 (Figure S.4).

When $y_c \geq 1$, the income cutoff lies beyond the possible income range ($0 \leq y \leq 1$), therefore the net change in cigarette expenditures will be fully progressive. This means that across the full income range ($0 \leq y \leq 1$), the ratio of net cigarette expenditures relative to income will increase with income.

When $y_c < 1$, the income cutoff lies within the possible income range ($0 \leq y \leq 1$), thus the net change in cigarette expenditures will be partially progressive. This means that for incomes y such that $0 \leq y \leq y_c$ the ratio of net cigarette expenditures relative to income will increase with income; while for incomes y such that $y_c < y \leq 1$ the ratio of net cigarette expenditures relative to income will decrease with income. On Figure S.4, the blue plateau separates the parameter spaces $(\varepsilon_g; s_g)$

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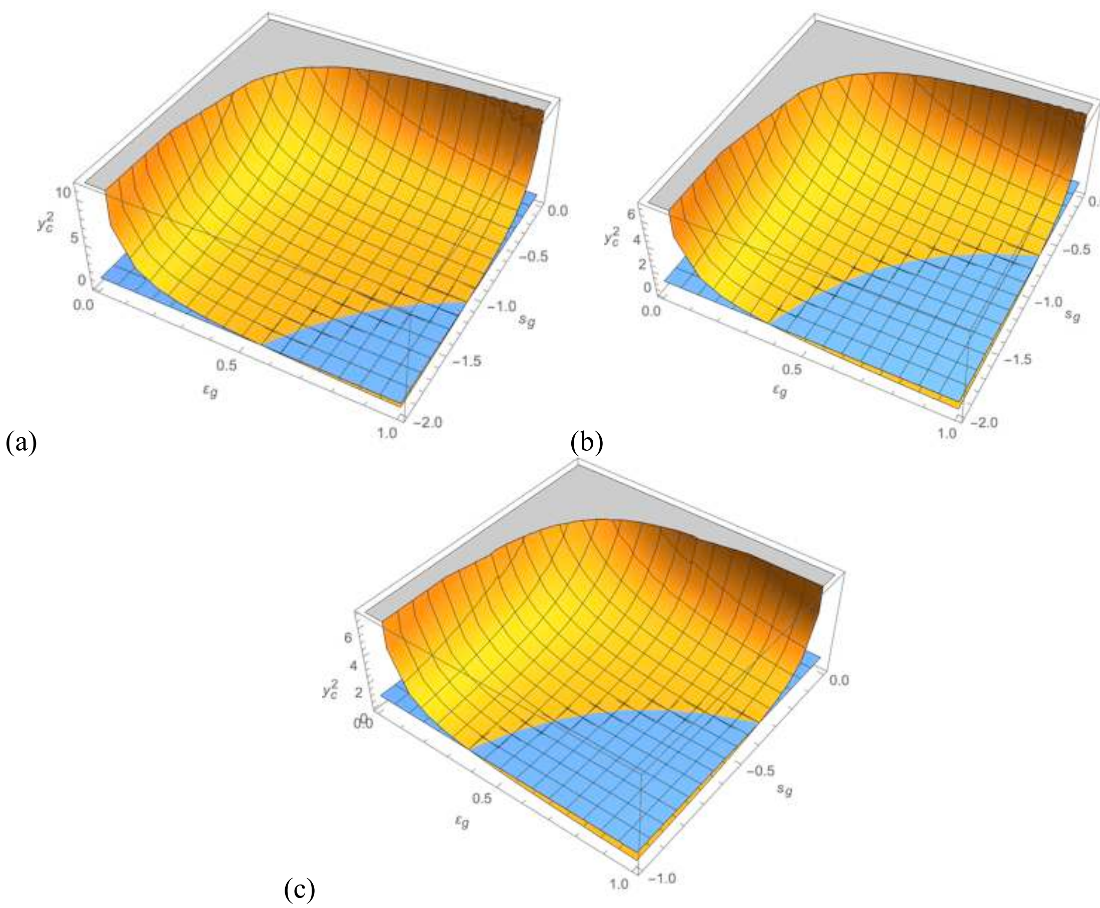
for which $y_c \geq 1$ (fully progressive net expenditures) and for which $y_c < 1$ (partially progressive net expenditures).

Figure S.4. Value of the cutoff function ${y_c}^2 = s_0 \frac{1+\varepsilon_0(r+1)}{\varepsilon_g s_g(r+1)}$ (yellow-orange surface) as a function of income gradient in price elasticity (ε_g) and income gradient in total cigarette consumption (s_g), for the following parameter values: price elasticity for the poorest $\varepsilon_0 = -1.00$; and relative price increase $r = 0.60$. The blue plateau separates the parameter spaces ($\varepsilon_g; s_g$) for which $y_c \geq 1$ (fully progressive net cigarette expenditures) and $y_c < 1$ (partially progressive net cigarette expenditures).

Different values of smoking consumption among the poorest individuals s_0.
(a) $s_0 = 0.30 * 10$ (smoking prevalence of 30% and daily consumption of 10 cigarettes);
(b) $s_0 = 0.20 * 10$ (smoking prevalence of 20% and daily consumption of 10 cigarettes);
(c) $s_0 = 0.10 * 10$ (smoking prevalence of 10% and daily consumption of 10 cigarettes);



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As a result, for different values of s_0 and for different ranges of values of ε_g and s_g, we are able to categorize the full vs. partial progressivity nature of the net change in cigarette expenditures, in identifying where the income cutoff y_c falls vis-à-vis 1 (Table S.4).

Table S.4. Categorization of progressivity of the net change in cigarette expenditures following increased tobacco taxation.
FP indicates full progressivity: across the full income range ($0 \leq y \leq 1$), the ratio of net cigarette expenditures relative to income will increase with income.
PP indicates partial progressivity: for incomes y such that $0 \leq y \leq y_c$ the ratio of net cigarette expenditures relative to income will increase with income; while for incomes y such that $y_c < y \leq 1$ the ratio of net cigarette expenditures relative to income will decrease with income.
The following input parameter values are used: price elasticity for the poorest $\varepsilon_0 = -1.00$; and relative price increase $r = 0.60$.

Different values of smoking consumption among the poorest individuals s_0.

(a) $s_0 = 0.30 * 10$ (smoking prevalence of 30% and daily consumption of 10 cigarettes).

$s_g \setminus \varepsilon_g$	**1.0**	**0.6**	**0.2**
-2.0	PP (until around $y_c = 0.8$)	FP	FP
-1.0	FP	FP	FP
-0.5	FP	FP	FP

(b) $s_0 = 0.20 * 10$ (smoking prevalence of 20% and daily consumption of 10 cigarettes).

$s_g \setminus \varepsilon_g$	**1.0**	**0.6**	**0.2**
-2.0	PP (until around $y_c = 0.6$)	PP (until around $y_c = 0.8$)	FP
-1.0	PP (until around $y_c = 0.9$)	FP	FP
-0.5	FP	FP	FP

(c) $s_0 = 0.10 * 10$ (smoking prevalence of 10% and daily consumption of 10 cigarettes).

$s_g \setminus \varepsilon_g$	**1.0**	**0.6**	**0.2**
-1.0	PP (until around $y_c = 0.6$)	PP (until around $y_c = 0.8$)	FP
-0.5	PP (until around $y_c = 0.9$)	FP	FP

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5. Country case studies

We applied our mathematical model to a number of country case studies including specific populations, time periods, cigarette retail prices and tax regimes, which covered a parameter space drawn from empirical situations and reasonable assumptions (the Philippines, Colombia, Bulgaria, Sweden, and the UK; see Table 3, main text).

For the Philippines, a relative price increase of $r = \frac{0.26}{0.36} = 0.72$ was observed over 2012-2014. With a price elasticity of demand among the poorest of $\varepsilon_0 = -0.87$, we would fulfill the first necessary condition of r and ε_0 being both sufficiently large as we would obtain: $1 + \varepsilon_0(r + 1) = -0.50 < 0$ (section 3.2 above, page 10). We would then see progressivity in the net change in cigarette expenditures. Moreover, with aggregate smoking prevalence of 22.4% and daily cigarette consumption of 13.8 (Table 3), we could conservatively assume smoking consumption among the poorest of $s_0 = 3.09$; assuming also 1.5 times greater consumption among the poorest than the richest (within the 1.2-1.6 times average ratio for the Western Pacific Region; see Table 2), we could conservatively derive consumption gradients of $s_g = -1.03$. Therefore, we would likely observe full progressivity of the net change in cigarette expenditures across the full income spectrum (see Figure S4 and Table S4; section 4 above).

For Colombia, a relative price increase of $r = \frac{0.51}{0.88} = 0.58$ was observed over 2016-2018. With a price elasticity of demand among the poorest of $\varepsilon_0 = -0.51$, we would not fulfill the necessary condition of r and ε_0 being both sufficiently large as we would obtain: $1 + \varepsilon_0(r + 1) = 0.19 > 0$. We would then see regressivity in the net change in cigarette expenditures.

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For Bulgaria, a relative price increase of $r = \frac{0.80}{1.60} = 1.00$ was observed over 2005-2014. With a price elasticity of demand among the poorest of $\varepsilon_0 = -1.33$, we would fulfill the necessary condition of r and ε_0 being both sufficiently large as we would obtain: $1 + \varepsilon_0(r + 1) = -1.66 < 0$. We would then see progressivity in the net change in cigarette expenditures. Moreover, with aggregate smoking prevalence of 36.0% and daily cigarette consumption of 9.8 (Table 3), we could conservatively assume smoking consumption among the poorest of $s_0 = 3.51$; assuming also 1.5 times greater consumption among the poorest than the richest (within the 1.3-1.6 times average ratio for the Europe Region; see Table 2), we could conservatively derive consumption gradients of $s_g = -1.17$. Therefore, we would likely observe full progressivity of the net change in cigarette expenditures across the full income spectrum (see Figure S4 and Table S4; section 4 above).

For Sweden, a relative price increase of $r = \frac{1.44}{4.93} = 0.29$ was observed over 2005-2014. With a price elasticity of demand among the poorest of $\varepsilon_0 = -0.50$, we would not fulfill the necessary condition of r and ε_0 being both sufficiently large as we would obtain: $1 + \varepsilon_0(r + 1) = 0.36 > 0$. We would then see regressivity in the net change in cigarette expenditures.

For the UK, a relative price increase of $r = \frac{1.61}{7.87} = 0.20$ was observed over 2005-2014. With a price elasticity of demand among the poorest of $\varepsilon_0 = -0.50$, we would not fulfill the necessary condition of r and ε_0 being both sufficiently large as we would obtain: $1 + \varepsilon_0(r + 1) = 0.40 > 0$. We would then see regressivity in the net change in cigarette expenditures.

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6. Incorporation of tobacco addiction into the mathematical model

We denote $\omega_a(y)$ the fraction of smokers of a given income y who are most addicted to tobacco and thus would not quit smoking or reduce tobacco consumption upon increased taxation δt.

As a result, before increased taxation, at the population level, smoking consumption across income y could be re-written as:

$$S_1(y) = \omega_a(y) * S_1(y) + \left(1 - \omega_a(y)\right) * S_1(y), \quad \text{(S.16)}$$

that is to say total smoking consumption is the sum of consumption from addicted smokers and of consumption from non-addicted smokers. After increased taxation, given that addicted smokers do not quit or reduce consumption, we would obtain:

$$S_2(y) = \omega_a(y) * S_1(y) + \left(1 - \omega_a(y)\right) * S_1(y) * [1 + \frac{\delta t}{p_1}\varepsilon(y)], \quad \text{(S.17)}$$

which would lead to the following net change in additional taxes (denoted ΔT_a):

$$\Delta T_a = S_1(y)[t_1 * p_1 * \frac{\delta t}{p_1}\varepsilon(y) + \delta t * \left[1 + \frac{\delta t}{p_1}\varepsilon(y)\right]] - \omega_a(y) * S_1(y)[(t_1 * p_1 + \delta t) * \frac{\delta t}{p_1}\varepsilon(y)].$$

(S.18)

Therefore, using $\Delta T(y) = T_2(y) - T_1(y) = S_1(y) * \frac{\delta t}{p_1} * [p_1 + \varepsilon(y) * (t_1 * p_1 + \delta t)]$ (equation 3 in the main text; base case scenario), we can rewrite ΔT_a as:

$$\Delta T_a = \Delta T - \omega_a(y) * S_1(y)[(t_1 * p_1 + \delta t) * \frac{\delta t}{p_1}\varepsilon(y)]. \quad \text{(S.19)}$$

Similarly, for the net change in cigarette expenditures, we would obtain:

$$\Delta C_a = \Delta C - \omega_a(y) * S_1(y)[(p_1 + \delta t) * \frac{\delta t}{p_1}\varepsilon(y)]. \quad \text{(S.20)}$$

Clearly, we have: $-\omega_a(y) * S_1(y)\left[(t_1p_1 + \delta t) * \frac{\delta t}{p_1}\varepsilon(y)\right] > 0$ and $-\omega_a(y) * S_1(y)\left[(p_1 + \delta t) * \frac{\delta t}{p_1}\varepsilon(y)\right] > 0$, because $\varepsilon(y) < 0$ and $\omega_a(y) > 0$. Thus: $\Delta T_a > \Delta T$ and $\Delta C_a > \Delta C$.

We can then directly see the impact of addiction varying with income y on the net change in additional taxes and net change in cigarette expenditures. When $\omega_a(y)$ decreases with income y increasing, we see how ΔT_a and ΔC_a could increase with the supplemental term $-\omega_a(y) * S_1(y)\left[(p_1 + \delta t) * \frac{\delta t}{p_1}\varepsilon(y)\right] > 0$ for lower income individuals, and hence reduce the progressivity potential of increased tobacco taxation for both net cigarette taxes and net cigarette expenditures.