A Population Dynamics Model of the Consequences of Menthol Cigarettes for Smoking Prevalence and Disease Risks

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This document describes the constructs of, and results from, the model commissioned by the Tobacco Products Scientific Advisory Committee (TPSAC) to estimate the consequences of menthol cigarette smoking on the U.S population. The model is an extension and modification of a population dynamics model previously developed to track smoking prevalence and smoking related risks, which has been extensively discussed in the literature.1-7 The following figure shows the general organization of the model, as modified to address menthol cigarettes:

**Menthol Model Block Simulation Diagram**

The boxes (compartments) represent the stock of individuals in different categories at a given time; the arrows represent the flow between compartments; and the circles represent
parameters that modify the flow. Red circles refer to parameters related to menthol smoking while green circles refer to the other parameters. Diamonds represent the event of smoking initiation, concentrated at a single age.

Following is a description of the constructs of the model:

**Definition of dynamic (time-dependent) variables:**

\[ P(a, t) = \text{US population of age } a \text{ in year } t \]

\[ N(a, t) = \text{Population of never – smokers of age } a \text{ in year } t \]

\[ F(a, t, q) = \text{Population of former – smokers of age } a, \text{ in year } t, \text{ that quit } q \text{ years ago} \]

\[ C(a, t) = \text{Population of current – smokers of age } a \text{ in year } t \]

\[ C_m(a, t) = \text{Population of current menthol – smokers of age } a \text{ in year } t \]

\[ C_n(a, t) = \text{Population of current non – menthol – smokers of age } a \text{ in year } t \]

\[ \pi_N(a, t) = \text{Prevalence of never – smokers of age } a \text{ in year } t \]

\[ \pi'_N(t) = \text{Adult prevalence of never – smokers in year } t \]

\[ \pi_F(a, t) = \text{Prevalence of former – smokers of age } a \text{ in year } t \]

\[ \pi'_F(t) = \text{Adult prevalence of former – smokers in year } t \]

\[ \pi_C(a, t) = \text{Prevalence of current – smokers of age } a \text{ in year } t \]

\[ \pi'_C(t) = \text{Adult prevalence of current – smokers in year } t \]

\[ \pi_{cm}(a, t) = \text{Prevalence of current menthol – smokers of age } a \text{ in year } t \]

\[ \pi'_{cm}(t) = \text{Adult prevalence of current menthol – smokers in year } t \]

\[ \pi_{cn}(a, t) = \text{Prevalence of current non – menthol – smokers of age } a \text{ in year } t \]

\[ \pi'_{cn}(t) = \text{Adult prevalence of current non – menthol – smokers in year } t \]

\[ D(t) = \text{Total deaths in year } t \]
**Definition of Non-dynamic variables and parameters:**

\( \mu(a) = \text{Overall death rate for individuals of age } a \)

\( \mu_N(a) = \text{Death rate among non-smokers of age } a \)

\( \mu_F(a,q) = \text{Death rate among former-smokers of age } a \text{ who quit } q \text{ years ago} \)

\( \mu_C(a) = \text{Death rate among current-smokers of age } a \)

\( \mu_{cm}(a) = \text{Death rate among current menthol-smokers of age } a \)

\( \mu_{cn}(a) = \text{Death rate among current non-menthol-smokers of age } a \)

\( \rho(a) = \text{Overall smoking quit rate for individuals of age } a \)

\( \rho_{cm}(a) = \text{Smoking quit rate for menthol-smokers of age } a \)

\( \rho_{cn}(a) = \text{Smoking quit rate for non-menthol-smokers of age } a \)

\( S_{m2n} = \text{Switching rate from menthol to non-menthol among current menthol smokers} \)

\( S_{n2m} = \text{Switching rate from non-menthol to menthol among current menthol smokers} \)

\( I = \text{Smoking initiation age} \)

\( \gamma = \text{Overall smoking initiation rate} \)

\( \gamma_{cm} = \text{Smoking initiation rate for menthol-smokers} \)

\( \gamma_{cn} = \text{Smoking initiation rate for non-menthol-smokers} \)

\( RR(a,q) = \text{Relative risk of death for a former smoker of age } a \text{ who quit } q \text{ years ago} \)

\( -q = 0 \text{ implies current smoker} \)

\( K_1 = \text{Mortality risk ratio} \left( \frac{\text{Menthol}}{\text{Non-menthol}} \right) \)

\( K_2 = \text{Quit rates ratio} \left( \frac{\text{Menthol}}{\text{Non-menthol}} \right) \)

\( K_3 = \text{Proportion of Menthol among Initiators} \)

\( K_4 = \text{Proportion of Menthol among Experimenters} \)
Supplemental material

Dynamic (time-dependent) relationships:

\[ K_5 = \text{Ratio of Yields from Experimenter to Established Smoker} \left( \frac{\text{Menthol}}{\text{Non-Menthol}} \right) \]

\[ N(0, t) = P(0, t) \]

\[ N(a, t) = N(a - 1, t - 1) \times (1 - \mu_N(a)) \text{ for } a \neq 1 \]

\[ N(a, t) = N(a - 1, t - 1) \times (1 - \mu_N(a)) \times (1 - \gamma_{cm} - \gamma_{cn}) \text{ for } a = 1 \]

\[ F(a, t, q) = 0 \text{ for } a - q \leq 1 \]

\[ F(a, t, 1) = C_m(a - 1, t - 1) \times (1 - \mu_{cm}(a - 1)) \times \rho_{cm}(a - 1) \]
\[ + C_n(a - 1, t - 1) \times (1 - \mu_{cn}(a - 1)) \times \rho_{cn}(a - 1) \text{ for } a - q > 1 \]

\[ F(a, t, q) = F(a - 1, t - 1, q - 1) \times (1 - \mu_{cp}(a - 1, q - 1)) \text{ for } a - q > 1 \text{ and } q > 1 \]

\[ C_m(a, t) = 0 \text{ for } a < 1 \]

\[ C_m(a, t) = \gamma_{cm} \times N(a - 1, t - 1) \times (1 - \mu_N(a - 1)) \text{ for } a = 1 \]

\[ C_m(a, t) = C_m(a - 1, t - 1) \times (1 - \mu_{cm}(a - 1)) \times (1 - \rho_{cm}(a - 1)) \times (1 - S_{m2n}(a - 1)) \]
\[ + C_n(a - 1, t - 1) \times (1 - \mu_{cn}(a - 1)) \times (1 - \rho_{cn}(a - 1)) \times S_{n2m} \text{ for } a > 1 \]

\[ C_n(a, t) = 0 \text{ for } a < 1 \]

\[ C_n(a, t) = \gamma_{cn} \times N(a - 1, t - 1) \times (1 - \mu_N(a - 1)) \text{ for } a = 1 \]

\[ C_n(a, t) = C_n(a - 1, t - 1) \times (1 - \mu_{cn}(a - 1)) \times (1 - \rho_{cn}(a - 1)) \times (1 - S_{n2m}(a - 1)) \]
\[ + C_m(a - 1, t - 1) \times (1 - \mu_{cm}(a - 1)) \times (1 - \rho_{cm}(a - 1)) \times S_{m2n} \text{ for } a > 1 \]

\[ P(a, t) = N(a, t) + \sum_{q=1}^{q=30} F(a, t, q) + C_m(a, t) + C_n(a, t) \]

\[ \pi_N(a, t) = \frac{N(a, t)}{P(a, t)} \]

\[ \pi'_N(t) = \frac{\sum_{a=18}^{a=100} N(a, t)}{\sum_{a=18}^{a=100} P(a, t)} \]
\[ \pi_F(a, t) = \frac{\sum_{q=1}^{30+} F(a, t, q)}{P(a, t)} \]
\[ \pi'_F(t) = \frac{\sum_{a=18}^{100} \sum_{q=1}^{30+} F(a, t, q)}{\sum_{a=18}^{100} P(a, t)} \]
\[ \pi_{c_m}(a, t) = \frac{C_m(a, t)}{P(a, t)} \]
\[ \pi'_{c_m}(t) = \frac{\sum_{a=18}^{100} C_m(a, t)}{\sum_{a=18}^{100} P(a, t)} \]
\[ \pi_{c_n}(a, t) = \frac{C_n(a, t)}{P(a, t)} \]
\[ \pi'_{c_n}(t) = \frac{\sum_{a=18}^{100} C_n(a, t)}{\sum_{a=18}^{100} P(a, t)} \]
\[ D(t) = \sum_{a=0}^{a=100} N(a, t) \times \mu_N(a) + \sum_{a=0}^{a=100} \sum_{q=1}^{30+} F(a, t, q) \times \mu_F(a, q) + \sum_{a=0}^{a=100} C_m(a, t) \times \mu_{c_m}(a) \]
\[ + \sum_{a=0}^{a=100} C_n(a, t) \times \mu_{c_n}(a) \]

**Non-dynamic relationships:**

- Expressions related to mortality risks and derivation of death rates for current-, former- and never-smokers given overall death rates \( \mu(a) \) in 2010.

\[ K_1 = \frac{\mu_{c_m}(a)}{\mu_{c_n}(a)} \]
\[ \mu_F(a, q) = \mu_N(a) \times RR(a, q) \]
\[ \mu_{c_m}(a) = K_1 \times \mu_N(a) \times RR(a, 0) \]
\[ \mu_{c_n}(a) = \mu_N(a) \times RR(a, 0) \]
\[ \mu(a) = \mu_N(a) \times \pi_N(a, 2010) + \left( \sum_{q=1}^{30+} \mu_N(a) \times RR(a, q) \times \pi_F(a, 2010, q) \right) \]
\[ + K_1 \times \mu_N(a) \times RR(a, 0) \times \pi_{cm}(a, 2010) \]
\[ + \mu_N(a) \times RR(a, 0) \times \pi_{cn}(a, 2010) \rightarrow \]
\[ \mu_N(a) = \frac{\mu(a)}{\pi_N(a, 2010) + \sum_{q=1}^{30+} (RR(a, q) \times \pi_F(a, 2010, q)) + K_1 \times RR(a, 0) \times \pi_{cm}(a, 2010) + RR(a, 0) \times \pi_{cn}(a, 2010)} \]

Expressions related to quit rates and derivation of quit rates for menthol and non-menthol smokers given overall quit rates \( \rho(a) \) in 2010.

\[ K_2 = \frac{\rho_{cm}(a)}{\rho_{cn}(a)} \]
\[ \rho_{cm}(a) = K_2 \times \rho_{cn}(a) \]
\[ \rho(a) = K_2 \times \rho_{cn}(a) \times \pi_{cm}(a, 2010) + \rho_{cn}(a) \times \pi_{cn}(a, 2010) \rightarrow \]
\[ \rho_{cn} = \frac{\rho(a)}{K_2 \times \pi_{cm}(a, 2010) + \pi_{cn}(a, 2010)} \]

• Expressions related to the initiation rate and derivation of initiation rate under the counterfactual scenario (in which menthol cigarettes do not exist) given overall smoking initiation rate \( \gamma \) in 2010.

\[ \gamma = \gamma_{cm} + \gamma_{cn} \]
\[ \gamma_{cm} = K_3 \times \gamma \]
\[ \gamma_{cn} = (1 - K_3 \times \gamma) \]
Let $W$ be the size of a cohort of potential experimenters, $E$ the proportion of experimenters in that cohort, $Y_m$ the proportion of menthol experimenters that become established smokers, and $Y_n$ the proportion of non-menthol experimenters that become established smokers; then, $W \times E \times K_4$ is the number of menthol experimenters and $W \times E \times (1 - K_4)$ is the number of non-menthol experimenters.

It follows that:

$$W \times E \times K_4 \times Y_m + W \times E \times (1 - K_4) \times Y_n = W \times \gamma$$

Given that $\frac{Y_m}{Y_n} = K_5$, then

$$W \times E \times K_4 \times K_5 \times Y_n + W \times E \times (1 - K_4) \times Y_n = W \times \gamma \quad \text{or}$$

$$Y_n = \frac{\gamma}{E \times (K_4 \times K_5 + (1 - K_4))}$$

Let $\gamma'$ be the initiation rate under the counterfactual, then, assuming the same proportion of experimenters as in the status-quo scenario:

$$W \times E \times Y_n = W \times \gamma' \quad \text{or}$$

$$\gamma' = E \times Y_n = \frac{E \times \gamma}{E \times (K_4 \times K_5 + (1 - K_4))} = \frac{\gamma}{K_4 \times K_5 + (1 - K_4)}$$