Evaluation of the Economic Impact of California's Tobacco Control Program: A Dynamic Model Approach--Appendix 2: A dynamic, normally distributed survival analysis of the relationship between aging, smoking history, and the mortality of men.

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Introduction

In this appendix, I derive a survival model that makes use of the expressions for the index of tobacco-exposure resulting from an individual's smoking history-derived in Appendix 1. Based on this survival model, expressions for the probability of living and dying are derived for never-smokers, current smokers, and former-smokers. The likelihood function for a sample of individuals based on these probabilitieis serves to estimate the parameters of the survival model, which include the parameters of the tobacco-exposure index..

Generally, summarizing smoking history with the tobacco exposure index, and the calculations that estimate the effect of smoking on morbidity, health status and medical costs that derive from usage of this index to summarize smoking history make three improvements over smoking status as the operative description of the effect of smoking behavior on health outcomes. The first improvement focuses on the level of information about an individual's smoking history. This exercise allows for greater detail about the relationship between variations in smoking behavior and about their causal effect on health outcomes. Details about an individual's smoking history can be incorporated into the measure used to summarize an individual's smoking behavior, the level of accumulated tobacco-exposure of an ever-smoker. The measure permits any combination of starting and stopping smoking times and any daily dossage level, measured as packs of cigarettes smoked per day.

The second improvement focuses on the causal effect of smoking on the deterioration of health outcomes. This improvement is meant to address the fact that estimates of the smoking attributable medical services are often greater for former smokers than they are for current smokers. In theory, this should not be the case. In this analysis, the derived measure of smoking's ability to damage health, the index of tobacco-exposure, incorporates theoretical distinctions between current and former-smokers that cause the expected damage to be less for former-smokers compared to current smokers, given all other dimensions of smoking history are the same. The effects due to smoking status, especially current-smoker versus former-smoker, of the relationship between smoking behavior and health outcomes that are to be estimated based on this model are not a "curve fit" exercise that best describes the smoking status-health outcome data. Rather, the estimates best fit the relationship between the effects of smoking behavior and health outcome expressed by the theory expressed in Appendix 1. In that theory the process describing tobacco-exposure implies that toxin levels fall when a current-smoker quits his smoking habit. If a former-smoker has his actual costs greater than a current-smoker, it results from the randomness in life, or the randomness in response to tobacco.

The third improvement focuses on the sample selection bias that is always present in analyses of the effect of smoking on health outcomes. Recognize that analyses of the health effects of smoking are performed on living populations. Death causes sample selection bias among living cohorts--alive responders are always the stronger members of any original cohort because they are the group least affected by smoking behavior. Consequently, the "all other things equal" assumption between never-smokers and ever-smokers is never met. Because the propensity to die for smokers is higher, the sample of smokers who remain alive is always inherently stronger than the sample of alive never-smokers. Thus, the estimated negative effects of smoking on health outcomes are always understated.

The method developed here is best described as a dynamic normally distributed survival analysis; or, perhaps, a dynamic Probit model. Rather than estimate the probability of an event occurring over a defined period of time, as in the Probit model, the dynamic normal survival model estimates the probability of an event occurring over an open ended, unfolding period of time. In this survival model: (1) the event of interest--in this case death--either occurs or it does not occur; (2) the propensity for the event to have occurred by time w is specified as equal to the expected value of the propensity of the event plus a random error term; (3) the variables specifying the expected value can vary continuously with time; (4) the error term at time w has a normal distribution, with (5) an expected value equal to zero, and (6) a variance that can vary with time.

If a respondent is an ever-smoker, his tobacco-exposure level is specified as equal to its expected value plus a random error. The expected value of tobacco-exposure, and the distribution of the random errors (the difference between the true value and the expected value) were derived in Appendix 1. Since the random error has a Normal distribution, the method and specification of the empirical analysis explained here is built on survival analyses that are based on random errors that have a Normal distribution.

A survival analysis (Kalbfleisch & Prentice, 1980) is developed in this Normal framework. The particular survival analysis developed here is of particular interest because it melds two historic lines of quantitative methods: limited dependent variable methods, which have been extensively developed by econometricians (Maddala, 1983), and survival analysis methods, which have been extensively developed by demographers, biostatisticians/epidemiologists, and engineers. In Section 2, I analytically construct a dynamic survivor model from a Probit like model describing the propensity to be dead at a particular time w in the random life span indicated by the variable T of a respondent. The propensity to die is specified as a linear sum of the expected value of an individual's propensity to experience the criterion event and a normally distributed random error. As in a survival model, the model describes the distribution of a respondent's life span ("time to failure"). The dynamic character of the resulting analysis is apparent in two ways. Rather than focusing on whether death {occurred, did not occur} over a defined, fixed period of time, as in the Probit model, the period under analysis is increasing with the passage of time, as in a survival analysis. Thus the Probit like specification of the propensity to be dead at each moment of time is transformed into a survival analysis describing the random life span variable T. This transformation is accomplished in the relationship between the propensity to be dead and the hazard rate, the instantaneous rate of failure (also known as the force for mortality and the failure rate) at each moment w.

Variations among parametric survival models focus on the functional form translating a model's hazard rate into its survival function, the models description of the probability that a respondent will live at least until time T. In the various models used in practice, hazard rates are either constants (such as in the exponential model, (Chiang, 1980)), functions of constants and powers of time (such as in the Weibull model,1939), multivariate--weighted linear sums of fixed characteristics (Tuma, Hannan, & Groeneveld, 1980), characteristics that vary at discrete points of time (Petersen, 1986a, 1986b), or, to a limited degree, characteristics that can be functions of time (Cox, 1972). All of the standard models (that I am aware of) yield closed form expressions for survivor functions and probability density functions of T. The analysis developed here makes use of technological and software developments. The analytically challenging event probability expressions are derived using *Mathematica* (*Mathematica*, Version 7.1, 2008). In the present analysis, the probability expressions for the never-smoker are closed form expressions-,but the probability expressions for current and former smokers are not and numerical integration methods must be used in the estimation of the model's coefficients.

In the analysis to follow, the determinants of the expected value of the propensity die at moment w, denoted by g[w], and the standard error of the random term of this propensity, denoted by σ[w], are functions of time and of parameters describing the cigarette smoking tobacco-exposure process. The analysis in Section 2 focuses on melding the Probit and Survival analyses. General probabilistic expressions for the observed sample events are obtained; that is, for the survivor function--the probability that a life span exceeds the time of data collection (a right censored event), and the probability density function of the life span T at moment of death t. To render these probability expressions applicable to the problem at hand, more detailed specifications are required before it is possible to construct the likelihood function for the observed sample. Section 3 presents a set of background comments that relate to how the specifications are to be made.

While the age of a respondent is observed, if the respondent is an ever-smoker, his accumulated tobacco-exposure is not observed. In Appendix 1, I presented the development of expressions for the theoretical distribution of tobacco-exposure of ever smokers. To render this Appendix "self-contained", a summary of the relevant closed form expressions is contained in Section 3. The tobacco-exposure distribution depends on: (1) an individual's smoking behavior (when smoking was initiated, what was its intensity (packs per day smoked), if and when did a respondent quit); (2) on parameters describing the distribution of tobacco-exposure, which require estimation; and (3) on randomness that is internal to the smoking process (depth of inhalation, an individual's inherent reaction to tobacco-toxin ingestion, variation in toxins per pack by brand, etc.). These tobacco-exposure effects are present in the propensity to die for ever-smokers. Second-hand smoke is not considered in this study. The expected level of tobacco-exposure is incorporated into the specification of the expected propensity to die by time w (g[w]); the randomness associated with a respondent's smoking history is incorporated into the random error of the propensity to die for ever-smokers, and consequently, effects the standard error of the propensity to die, σ[w], of respondants in ever-smoker groups. The random errors in the propensity to die for ever-smokers include both the random error describing everyone's random chance in life (the random error in the never-smokers propensity to die equation) and the random error describing an individual's random response to smoking. For every smoking history group, the resulting random error in the propensity to die has a Normal distribution (Kotz, Balakrishnan, & Johnson, 2000).

The specifications assume that never-smokers form the basis of the description between age and death for smokers and never-smokers. Based on the general probabilistic expressions developed in Section 2 and the specification of the model, the probability expressions developed for never-smokers, current-smokers, and former-smokers in Sections 4, 5, and 6. For never-smokers, the expected propensity to die by time w is specified in Section 4 as a linear function of age and age-squared as well as a random variable that increases with time. The basic randomness in the propensity to die for never-smokers is the randomness representing the vicissitudes of life. This randomness is also present in the normal random variable of the propensity to die for respondents who are ever-smokers. For current-smokers, the expected propensity to die by time w is specified in Section 5 as equal to the expected propensity to die by time w for the never-smoker plus a linear function of the current-smoker's tobacco-exposure. The random variable is equal to the random variable of the never-smoke, plus the product of the coefficient on the expected tobacco-toxin and the difference between the actual tobacco toxin level for the individual and the expected value of his tobacco exposure. This difference is a random variable whose variance was derived in Appendix 1. For former-smokers, the expected propensity to die by time w is specified in Section 6 as equal to the expected propensity to die by time w for the never-smoker, plus the expected propensity to die for current smokers by time te--the time the individual ended smoking--plus the expected value of the propensity to die for former smokers who have babstained from smoking for time u. The random variable has a component from each these expressions.

Section 2: A dynamic Normal survival model.

Let T represent a random variable denoting the life span of a respondent (time to failure) and let F[T<=w] denote the probability that a respondent will die prior to time w. F[T<=w] is the probability distribution of T. Let h[w] + O[△] denote the probability that an individual will die within the interval [w, w + △). h[w] denotes the rate of dying at time w. In the older literature h[w] is known as the "force of mortality" (Gompertz, 1825; Makeham, 1860); in later literature h[w] is known as the hazard rate or the failure rate (Kalbfleisch & Prentice, 1980). O[△] represents second order effects. O[△] is a function of △; it tends to zero faster than △ tends to zero (Chiang, 1980).

The modern theory of survivor analysis derives from the construction of the differential equation describing how the distribution of the life span T changes over time. To the best of my knowledge, this approach was first offered for the Poisson process by Feller (1957). If an individual dies prior to the time w + △, the probability of this event can be expressed by F[T<=w+△]. The respondent must either have died prior to w, with probability F[T<=w], or if he lived to time w, the event has a probabilty (1 - F[T<=t]), then he must have died between w and w + $\triangle$, with probability (h[w] + O[$\triangle$]). The probabilistic statement detailing these possibilities is given by equation [2.1],

$$[2.1] \quad F[T\le w+\triangle] = F[T\le w] + (1 - F[T\le w])(h[w] + O[\triangle]).$$

Rearranging terms (moving F[T<=w] to the left side of the equality), dividing through by $\triangle$, and taking the limit as $\triangle$ goes to zero yields the differential equation describing the time rate of change of the distribution of T. The probability density function of T (denoted by f[w]) follows from these operations and is given by equation [2.2a], where the distribution function is subject to the initial condition that it is equal to 0 when the process begins, F[T=0]=0. Equation [2.2.2b] represents this initial condition,

$$[2.2a] \quad f[w] = d/dw\, F[T\le w] = (1 - F[T\le w])\, h[w],$$

subject to

$$[2.2b] \quad F[T=0]=0.$$

The solution to equations [2.2a], subject to [2.2b], defines the survival function, the probability that time to death exceeds time w. This probability, denoted by G[T>w] is given by equation [2.3],

$$[2.3] \quad G[T>w] = (1 - F[T\le w]) = \mathrm{Exp}\left[-\int_0^w h[\tau]\, d\tau\right].$$

We begin by constructing the propensity of a respondent to be dead at some time w, $0 \leqslant w \leqslant t$. The propensity to be dead at w is denoted death*[w]. Assume that the propensity to be dead at w is the sum of the expected value of the propensity evaluated at time w, denoted by g[w], and a random error at time w, denoted by ς[w]. Whether the individual is dead or alive at time w (1 or 0, respectively) is a measure of the observable event "the observation is dead or alive at time w", respectively. If the propensity to be dead is greater than zero, an observed measure will be one, and vise-a-versa. Equation [2.4a], defines the propensity to be dead at time w. Equation [2.4b] defines the relationship between an individual's propensity score and his observable measure death[w]; equation [2.4c] defines the distribution of the random variable at time w, [2.4a] death*[w] = g[w] + ς[w];

where:

g[w] is the expected value at time w of the respondent's propensity to have died by time w;

ς[w] is a random variable at time w;

[2.4b] death*[w] {>, ⩽} 0, death[w] = {1,0},

and

[2.4c] ς[w] ~ Normal[0, σ^2[w]].

With the exception that a Probit model expresses equations [2.4a] through [2.4c] for a fixed interval of time rather than for a particular time w, equations [2.4a] through [2.4c] describe the Probit model, which perhaps suggests the Probit name for the survival model under development.

In survival analyses, the hazard rate is defined as the ratio of the rate of change of the probability of dying to the probability of being alive. With this propensity score, the maximum probability of being alive is measured by the distribution function evaluated at a propensity to die equal to the value zero. Time rates of change in this probability will also occur at this propensity value. The description of the propensity to be dead by time w implies that the propensity score has a normal distribution with a mean g[w] and a variance σ^2[w]. This distribution implies that equation [2.1] can be stated in Normal distribution terms as equation [2.5],

[2.5] (1 - Φ[(death*[w + △]-g[w + △])/σ[w + △]) =

(1 - Φ[(death*[w]-g[w])/σ[w]]) +

Φ[(death*[w]-g[w])/σ[w]] (h[w] + O[△]),

where Φ[] is the normal distribution function. Replicating the steps that led from equation [2.1] to equation [2.2] yields an expression for the hazard rate of this problem; that is--rearrange terms, divide by △, and take the limit as △ goes to zero--and then (1) evaluate the expressions at death*[w]=0, and (2) solve for the hazard rate, h[w]. Equation [2.6] describes the hazard rate at time w for this problem,

[2.6] $h[w] = \partial_w(\Phi[g[w]/\sigma[w]])/(1-\Phi[g[w]/\sigma[w]])$

$= \{(1/\sigma[w])\ \varphi[g[w]/\sigma[w]]\ (\partial_w\, g[w]\ /\ \sigma[w])\}\ /$

$(1-\Phi-g[w]/\sigma[w]]),$

where $\varphi[]$ is the normal probability density function and ∂_w denotes the partial derivative with respect to w.

The survival function, G[T>w], and the probability density function, f[T=w], of the random life-span variable T are, respectively, the probability that a respondent was alive when the data were collected at time w, and the probability that a respondent lived until time w, and then died at time w. These are the probabilities of the observed events that are associated with the life and death of the respondents under analysis. Based on the survival function and the hazard rate (equations [2.3] and [2.2a], above) the probability of survival and the probability density function expressions are given by equations [2.7a] and [2.7b],

[2.7a] $G[T>t] = (1 - F[T<=t]) = \mathrm{Exp}[-\int_0^t h[w]\, dw],$

and

[2.7b] $f[t] = d/dt\ F[T<=t] = G[T>t]\ h[t].$

The likelihood expression for a sample is the product of the probabilities associated with each of the observed events in a sample. Explicit development of the likelihood function for this problem requires further specification, which will begin to be made in Section 4. Section 3 presents background considerations that affect the specification of the model.

Section 3: Background considerations about time, tobacco-exposure, and heterogeneity.

It is useful to begin a discussion of the specific implementation of the model with background considerations about how time is notated and treated in the model.

The zero point of time is taken to be the mean age that American male's begin to smoke, 17 years of age (REF to NMES). Moreover, time is measured in decades. Thus the age of a 40 year old is measured with a time value of 2.3 decades, ((40 - 17)/10).

Prior to age 35 or 40 (depending on the specific disease) epidemiologists do not generally attribute negative effects of smoking behavior on health, especially its effect on smoking related diseases (Sammet, ????). Consistent with this framework, parameter estimations, both in the mortality model under discussion, and in the smoking related disease models (see Appendix 3), are based on respondents who are at least 40 years of age (2.3 decades in the age units used in the study's time measure).

Time has different relevant meanings within the different smoking statuses. The notation to be developed will account for all of these differences. More specifically, in the specification to be developed for never-smokers, time represents age; in the specification to be developed for current-smokers, time represents both age and time smoked; and in the specification to be developed for former-smokers, time represents age, the duration of time smoked, and the duration of time a respondent abstained from smoking. As equation [2.4.3] below will show, the expected value of the propensity to be dead at time w for never-smokers is specified as a linear function of age and age-squared. For current and former smokers, the specification of the propensity to be dead includes these same never-smoker terms. Additionally, the specification includes a coefficient weighted expected level of tobacco-exposure, which estimates the effect of smoking history on the propensity to die. The propensity to be dead also has a random variable and the variance of this random variable affects an ever-smoker's probability of dying. In every respondent's propensity to be dead, the random variable includes a term associated with the random variable in the never-smokers propensity to be dead. This term represents the general vicissitudes of life. For ever-smokers, additionally, the error term includes the product of the coefficient on the tobacco-exposure variable in the expected propensity to be dead and a random variable measuring the difference between a recipient's true tobacco-exposure level and his expected tobacco-exposure level, given his smoking history. Thus the variance of the random variable in an ever-smoker's propensity to be dead includes the square of the coefficient on the tobacco-exposure measure in the expected value of the ever-smoker's propensity and the variance of the difference between the true and expected tobacco-exposure in the body of the ever-smoker.

As depicted in equations [2.7a] and [2.7b] above, the notation used in the survival function and probability density functions, respectively, describe the observed events--lived between time 0 and w (where w represents the final observation), or died at time w, (here w represents time of death, which is after the acquisition of smoking history, but before the final observation about death in 1999. These probability expressions include exponentials of the integral of the negative of the hazard function over the relevant time period. Based on this data set, the integration is actually over the age of each respondent from the initial smoking history acquisition, to either the respondent's age at death, or his age when the final accumulation of death data was completed.

To represent both age, and duration of smoking (and for a former-smoker, period of abstention) in the same integration over observed time, I created a recipient specific coefficient "α" to represent a transformation of a recipient's decades of age into his decades of smoking duration. That is, "α" equals the difference between a respondent's duration of smoking and his age. Consequently, age + α equals duration of smoking. For current-smokers, the integration in the survival function occurs over the recipient's age (w) to his age at death or age at the time of final data collection (w + follow-up time). However, the time dimensions in a current-smoker's tobacco-toxin expression are measuring smoking time. Thus during the period under analysis the levels of tobacco-exposure are being evaluated for the years the respondent smoked; from (age + α) to (age + follow-up time + α). Similarly, the time dimensions in a former-smoker's tobacco-toxin expression are measuring decades of abstention, given decades smoked, and the levels of tobacco-exposure are being evaluated between the years the respondent had abstained at his age when observation started to the years the respondent had abstained when observation ceased. The decades a former-smoker smoked are denoted by te. The decades he subsequently abstained from smoking are denoted by u. The integration for former-smokers is over an abstention period (respondents are classified as former-smokers on their base-line interview). Thus the time they smoked, te, is a given, and the duration of abstention from smoking variable, u, expressed in terms of age as u = w + α - te, is integrated over the age of the respondent during his smoking abstention and either his age at time at death or at final data collection.

With temporal notation explained, it is possible to understand equations [3.1a] and [3.1b], closed form expressions describing the expected level and the variance, respectively, of the level of tobacco-exposure for a current-smoker at age w during the observation period in the NAS-NRC data. Equations [3.2a] and [3.2b] report these same expressions for former-smokers. The derivations of these expressions were made in Appendix 1. Here, these expressions are to be taken as given.

[3.1 a] $tox_c[w, \alpha]$ =

$$= \left\{\frac{1}{2\gamma 1}\left(e^{-(w+\alpha)\sqrt{\gamma 1}}\left(\left(-1+e^{(w+\alpha)\sqrt{\gamma 1}}\right)^2\gamma_0+\left(-1+e^{2(w+\alpha)\sqrt{\gamma 1}}\right)\sqrt{\gamma 1}\,(p\,\delta-\nu_{c0})\right)\right)\right\}+ \epsilon_{toxc}[w+\alpha];$$

where:

[3.1b] $$\epsilon_{toxc}[w+\alpha] \sim N\left[0, \left\{\frac{1}{\gamma 1}\left((w+\alpha)\,\text{Sinh}\left[(w+\alpha)\sqrt{\gamma 1}\right]^2\sigma_c^2\right)\right\}\right] = N[0, \sigma_{toxc}{}^2[t]].$$

[3.2 a] $tox_f[u, te]$ =

$$= \frac{1}{2\gamma 1}\left(e^{(-(te+u))\sqrt{\gamma 1}}\left(-2\,e^{(te+u)\sqrt{\gamma 1}}\,p\sqrt{\gamma 1}\,\delta\,\text{Sinh}\left[u\sqrt{\gamma 1}\right]+\left(-1+e^{(te+u)\sqrt{\gamma 1}}\right)^2\gamma_0+\left(-1+e^{2(te+u)\sqrt{\gamma 1}}\right)\sqrt{\gamma 1}\,(p\,\delta-\nu_{c0})\right)\right) + \epsilon_{tox_f}[u, te];$$

where:

[3.2b] $$\epsilon_{toxf}[u, te] \sim N\left[0, \frac{1}{\gamma 1}\left(\frac{1}{4}\,te\left(-2+\text{Cosh}\left[2(te-u)\sqrt{\gamma 1}\right]+\text{Cosh}\left[2(te+u)\sqrt{\gamma 1}\right]\right)\sigma_c^2+u\,\text{Sinh}\left[u\sqrt{\gamma 1}\right]^2\sigma_f^2\right)\right] =$$

$$N[0, \sigma_{toxf}{}^2[u,te]].$$

In a Probit model with a homogenous variance, the propensity equation is implicitly "standardized". The assumed error term's unit variance is achieved by implicitly dividing the propensity expression by the (unknown) standard error of the random error term. The implicit division renders the coefficients in the expected value "standardized" and the model with a random error that has a variance equal to one. In a Probit model with heterogenous variance, implicitly a similar step is taken. The heterogenous variance might be specified as the exponential of a weighted linear sum of characteristics, say Exp[Zγ]. Feasibility of parameter estimation requires that Z not have a column of one's, which would be multiplying an intercept term in the vector γ (Green, 1990). If γ0 were the coefficient on an intercept, and if the remaining part of the variance's Zγ description were partitioned to separate γ0 from the remaining products of gamma coefficients and their Z variables, the latter of which will be denoted by Z1γ1, then [Zγ]= Exp[γ0] Exp[Z1γ1]. The absence of Exp[γ0] is equivalent to having divided the specification of the propensity score by the square root of Exp[γ0] (i.e., Exp[γ0/2], to remove Exp[γ0] from the variance specification.

For the never-smokers in this study, the variance arising from integrating white noise over time, for example from age 40, measured as 2.3, to age 46, measured as 2.9, is $\sigma^2{}_n$ 0.6. The never-smoker's death propensity equation is divided by σ_n and the variance of the random error in the propensity of a never-smoker by age w is expressed as the value of his age, w. For current and former-smokers, additionally, the variance includes a term associated with the coefficient weighted variance in the distribution of tobacco-toxins in the body. Thus the coefficients on the variables in the expected level of the propensity to be dead by age w are "standardized" by the standard deviation in the vicissitudes of life. Additionally, the coefficients indicating the constants in the propensity to be dead for every smoking status, whose description is yet to be made, are similarly standardized.

- ### Section 4: The probability that a never-smoker lives longer than the final data collection date, or that he dies between the initial base-line and the final data collection date.

A never-smoker's age (measured in the units of the problem--decades, with zero equal to 17 years of age)--is represented by the variable w. I specify the expected value of the propensity to be dead by age w (or what would be age w if the person were alive) as the sum of the product of a constant, η1, and the individual's age (w), and the product of a second constant, η2, and the square of the individual's age$\left(w^2\right)$.

Equation [4.1] presents the relevant equation,

[4.1] $\text{death}^*{}_n[w] = \eta 1\ w + \eta 2\ w^2 + \varsigma_n[w]$,

where $\varsigma_n[w] \sim N[0, w]$.

A point to emphasize here, which is true for the specifications of the model for all smoking status groups, is that this is a dynamic model. All of the variables in the expected value of the propensity to be dead, as well as the variance of the error in the propensity to be dead are changing continuously as the respondent ages. With respect to the error term, ς_n[w], I have assumed that a white noise process underlies the random error expressed in the propensity to be dead. As commented on in Section 3 above, I have also assumed that the propensity specification has been standardized by the size of the standard deviation of the Brownian motion (white-noise) process. Hence, the eta coefficients are to be understood as standardized. It follows that the hazard-rate for the never-smoker at age w is given as follow:

The hazard-rate for a never-smoker, the ratio of the time rate of change in the probability of dying at age w, divided by the probability of living at age w, is given (in *Mathematica* notation) by equation [4.2],

[4.2]

$$h_n = D\left[\mathrm{CDF}\left[\mathrm{NormalDistribution}\left[0, \sqrt{w}\right], (\eta1\ w + \eta2\ w^2)\right], w\right] \Big/ \left(1 - \mathrm{CDF}\left[\mathrm{NormalDistribution}\left[0, \sqrt{w}\right], (\eta1\ w + \eta2\ w^2)\right]\right)$$

$$\frac{e^{-\frac{(w\,\eta1 + w^2\,\eta2)^2}{2\,w}} \left(\frac{\eta1 + 2\,w\,\eta2}{\sqrt{2}\,\sqrt{w}} - \frac{w\,\eta1 + w^2\,\eta2}{2\,\sqrt{2}\,w^{3/2}}\right)}{\sqrt{\pi}\left(1 + \frac{1}{2}\left(-1 - \mathrm{Erf}\left[\frac{w\,\eta1 + w^2\,\eta2}{\sqrt{2}\,\sqrt{w}}\right]\right)\right)}$$

The survival function for a never - smokes at age "ulag" (which is the upper limit of the integration over age), is the probability that the never - smoker will live beyond age = ulage, given that he was alive at a base - line age, llage (lower - limit of the integration over age). It is equal to the exponential of the integral of the negative of the hazard - rate over the period of observation, from llage, the age of the respondent at base - line, to ulage, the age of the respondent when observation is complete, which is either (1) when follow - up is completed; or (2) age at death. The Mathematica expression for the survival function is given by equation [4 .3],

[4 .3]

$$g_n =$$

$$\mathrm{Exp}\left[\mathrm{Integrate}\left[-\frac{e^{-\frac{(w\,\eta1 + w^2\,\eta2)^2}{2\,w}} \left(\frac{\eta1 + 2\,w\,\eta2}{\sqrt{2}\,\sqrt{w}} - \frac{w\,\eta1 + w^2\,\eta2}{2\,\sqrt{2}\,w^{3/2}}\right)}{\sqrt{\pi}\left(1 + \frac{1}{2}\left(-1 - \mathrm{Erf}\left[\frac{w\,\eta1 + w^2\,\eta2}{\sqrt{2}\,\sqrt{w}}\right]\right)\right)}, \{w, \mathrm{llagein66}, \mathrm{ulage}\}, \mathrm{Assumptions} \to \{\mathrm{Element}[\{\mathrm{llagein66}, \mathrm{ulage}\}, \mathrm{Reals}]\ \&\&\ \mathrm{llagein66} < \mathrm{ulage}\ \&\&\ \mathrm{llagein66} > 0\}\right]\right]$$

$$\frac{-1+\text{Erf}\left[\frac{\sqrt{\text{ulage}}\ (\eta 1+\text{ulage}\ \eta 2)}{\sqrt{2}}\right]}{-1+\text{Erf}\left[\frac{\sqrt{\text{llagein66}}\ (\eta 1+\text{llagein66}\ \eta 2)}{\sqrt{2}}\right]}$$

The hazard-rate evaluated at age of death is given by equation [4.4],

[4.4]

$$ht_n = h_n\ /.\ w \to \text{ulage}$$

$$\frac{e^{-\frac{(\text{ulage}\ \eta 1+\text{ulage}^2\ \eta 2)^2}{2\,\text{ulage}}}\left(\frac{\eta 1+2\,\text{ulage}\ \eta 2}{\sqrt{2}\ \sqrt{\text{ulage}}}-\frac{\text{ulage}\ \eta 1+\text{ulage}^2\ \eta 2}{2\sqrt{2}\ \text{ulage}^{3/2}}\right)}{\sqrt{\pi}\left(1+\frac{1}{2}\left(-1-\text{Erf}\left[\frac{\text{ulage}\ \eta 1+\text{ulage}^2\ \eta 2}{\sqrt{2}\ \sqrt{\text{ulage}}}\right]\right)\right)}$$

The probability density function of the random life span for a never smoker is the product of the hazard-rate evaluated at time of death and the survival function evaluated to the time of death, equation [4.5],

[4.5]

$$f_n = ht_n\ g_n$$

$$\left(e^{-\frac{(\text{ulage}\ \eta 1+\text{ulage}^2\ \eta 2)^2}{2\,\text{ulage}}}\left(\frac{\eta 1+2\,\text{ulage}\ \eta 2}{\sqrt{2}\ \sqrt{\text{ulage}}}-\frac{\text{ulage}\ \eta 1+\text{ulage}^2\ \eta 2}{2\sqrt{2}\ \text{ulage}^{3/2}}\right)\left(-1+\text{Erf}\left[\frac{\sqrt{\text{ulage}}\ (\eta 1+\text{ulage}\ \eta 2)}{\sqrt{2}}\right]\right)\right)\Big/$$
$$\left(\sqrt{\pi}\left(-1+\text{Erf}\left[\frac{\sqrt{\text{llagein66}}\ (\eta 1+\text{llagein66}\ \eta 2)}{\sqrt{2}}\right]\right)\left(1+\frac{1}{2}\left(-1-\text{Erf}\left[\frac{\text{ulage}\ \eta 1+\text{ulage}^2\ \eta 2}{\sqrt{2}\ \sqrt{\text{ulage}}}\right]\right)\right)\right)$$

Section 5: The dynamic Normal survival model for Current-Smokers.

The nomenclature in the distribution of tobacco-exposure, is as follows: The variable w is a measure of a respondent's age (in decades after age 17). The distribution of tobacco-exposure of current smokers at age w after smoking is initiated is Normal. Its expected value and variance are given by equation [5.1], (see Appendix 1 for its derivation):

[5.1]

$$\mathbf{tox_c[w + \alpha] \sim N\left[\left(\frac{1}{2\,\gamma1}\left(e^{-(w+\alpha)\sqrt{\gamma1}}\left(\left(-1 + e^{(w+\alpha)\sqrt{\gamma1}}\right)^2 \gamma0 + \left(-1 + e^{2\,(w+\alpha)\sqrt{\gamma1}}\right)\sqrt{\gamma1}\,(p\,\delta - \nu c0)\right)\right)\right), \left(\frac{(w+\alpha)\,\mathrm{Sinh}\left[(w+\alpha)\sqrt{\gamma1}\right]^2 \sigma c^2}{\gamma1}\right)\right]}$$

where:

w is the age of the respondent;

α is the difference between the time the respondant initiated smoking and his age (measured in the units of the problem, so that w + α is the duration a respondent smoked;

$\gamma0$ is the trend in the time rate of change of the purge rate;

$\gamma1$ is the marginal effect of a unit of tobacco-exposure on the time rate of change of the purge rate;

$\nu c0$ is the purge rate when smoking is initiated,

p is the packs of cigarettes smoked per day;

δ is the toxins per pack smoked, and

σc^2 is the square of the standard deviation of the Brownian motion process of the random variable in the specification of the time rate of change of the purge-rate. This Brownian motion process has been "standardized" by the standard deviation of the Brownian motion process of describing the vicitudes of life, which is the dynamic process leading to the propensity to die by time w for a never-smoker, and applies to all respondents.

Equation [5.2.a] is the expression for the expected value of $tox_c[w + \alpha]$,

[5.2a]

$$\mathbf{tox_c[w + \alpha] = \left(\frac{1}{2\,\gamma1}\left(e^{-(w+\alpha)\sqrt{\gamma1}}\left(\left(-1 + e^{(w+\alpha)\sqrt{\gamma1}}\right)^2 \gamma0 + \left(-1 + e^{2\,(w+\alpha)\sqrt{\gamma1}}\right)\sqrt{\gamma1}\,(p\,\delta - \nu c0)\right)\right)\right)}$$

$$\frac{e^{(-w-\alpha)\sqrt{\gamma1}}\left(\left(-1 + e^{(w+\alpha)\sqrt{\gamma1}}\right)^2 \gamma0 + \left(-1 + e^{2\,(w+\alpha)\sqrt{\gamma1}}\right)\sqrt{\gamma1}\,(p\,\delta - \nu c0)\right)}{2\,\gamma1}$$

and equation [5.2.b] is the expression for the time rate of change of the expected value of $tox_c[w+\alpha]$,

[5.2b]

$$tox_c'[w+\alpha] = D[tox_c[w+\alpha], w]$$

$$-\frac{e^{(-w-\alpha)\sqrt{\gamma 1}}\left(\left(-1+e^{(w+\alpha)\sqrt{\gamma 1}}\right)^2 \gamma 0 + \left(-1+e^{2(w+\alpha)\sqrt{\gamma 1}}\right)\sqrt{\gamma 1}\,(p\,\delta - \nu c0)\right)}{2\sqrt{\gamma 1}} + \frac{1}{2\,\gamma 1}$$

$$e^{(-w-\alpha)\sqrt{\gamma 1}}\left(2\,e^{(w+\alpha)\sqrt{\gamma 1}}\left(-1+e^{(w+\alpha)\sqrt{\gamma 1}}\right)\gamma 0\sqrt{\gamma 1} + 2\,e^{2(w+\alpha)\sqrt{\gamma 1}}\,\gamma 1\,(p\,\delta - \nu c0)\right)$$

Equation [5.2.c] is the expression for the standard deviation of the error term in the latent index of death, denoted $\sigma tox_c[w+\alpha]$,

[5.2c]

$$\sigma tox_c[w+\alpha] = Sqrt\left[w + \eta 3^2\left(\frac{(w+\alpha)\,Sinh\left[(w+\alpha)\sqrt{\gamma 1}\right]^2 \sigma c^2}{\gamma 1}\right)\right]$$

$$\sqrt{w + \frac{(w+\alpha)\,\eta 3^2\,\sigma c^2\,Sinh\left[(w+\alpha)\sqrt{\gamma 1}\right]^2}{\gamma 1}}$$

and equation [5.2.d] is the expression for the time rate of change in the standard deviation of latent index of death, denoted $\sigma tox_c[w+\alpha]$,

[5.2d]

$$\sigma tox_c'[w+\alpha] = D[\sigma tox_c[w+\alpha], w]$$

$$\left(1 + \frac{1}{\sqrt{\gamma 1}} 2\,(w+\alpha)\,\eta 3^2\,\sigma c^2\,Cosh\left[(w+\alpha)\sqrt{\gamma 1}\right]\right.$$

$$\left. Sinh\left[(w+\alpha)\sqrt{\gamma 1}\right] + \frac{\eta 3^2\,\sigma c^2\,Sinh\left[(w+\alpha)\sqrt{\gamma 1}\right]^2}{\gamma 1}\right) \Big/$$

$$\left(2\sqrt{w + \frac{(w+\alpha)\,\eta 3^2\,\sigma c^2\,Sinh\left[(w+\alpha)\sqrt{\gamma 1}\right]^2}{\gamma 1}}\right)$$

Equation [5.2.e] is the expression for the time rate of change in the probability of dying,

[5.2e]

D[CDF[NormalDistribution[0, σtox$_c$[w + α]], (η1 w + η2 w^2 + η3 tox$_c$[w + α])], w]

$$\frac{1}{\sqrt{\pi}} e^{-\frac{\left(w\,\eta 1+w^2\,\eta 2+\frac{e^{(-w-\alpha)\sqrt{\gamma 1}}\,\eta 3\left(\left(-1+e^{(w+\alpha)\sqrt{\gamma 1}}\right)^2\gamma 0+\left(-1+e^{2(w+\alpha)\sqrt{\gamma 1}}\right)\sqrt{\gamma 1}\,(p\,\delta-\nu c0)\right)}{2\,\gamma 1}\right)^2}{2\left(w+\frac{(w+\alpha)\,\eta 3^2\,\sigma c^2\,\mathrm{Sinh}\left[(w+\alpha)\sqrt{\gamma 1}\right]^2}{\gamma 1}\right)}} \left(-\left(\left(w\,\eta 1+w^2\,\eta 2+\frac{1}{2\,\gamma 1}\,e^{(-w-\alpha)\sqrt{\gamma 1}}\,\eta 3\left(\left(-1+e^{(w+\alpha)\sqrt{\gamma 1}}\right)^2\gamma 0+\left(-1+e^{2(w+\alpha)\sqrt{\gamma 1}}\right)\sqrt{\gamma 1}\,(p\,\delta-\nu c0)\right)\right)\right.\right.$$

$$\left(1+\frac{1}{\sqrt{\gamma 1}}\,2\,(w+\alpha)\,\eta 3^2\,\sigma c^2\,\mathrm{Cosh}\left[(w+\alpha)\sqrt{\gamma 1}\right]\,\mathrm{Sinh}\left[(w+\alpha)\sqrt{\gamma 1}\right]+\frac{\eta 3^2\,\sigma c^2\,\mathrm{Sinh}\left[(w+\alpha)\sqrt{\gamma 1}\right]^2}{\gamma 1}\right)\Bigg)\Bigg)\Big/$$

$$\left(2\sqrt{2}\left(w+\frac{1}{\gamma 1}(w+\alpha)\,\eta 3^2\,\sigma c^2\,\mathrm{Sinh}\left[(w+\alpha)\sqrt{\gamma 1}\right]^2\right)^{3/2}\right)+$$

$$\left(\eta 1+2\,w\,\eta 2-\frac{1}{2\sqrt{\gamma 1}}\,e^{(-w-\alpha)\sqrt{\gamma 1}}\,\eta 3\left(\left(-1+e^{(w+\alpha)\sqrt{\gamma 1}}\right)^2\gamma 0+\left(-1+e^{2(w+\alpha)\sqrt{\gamma 1}}\right)\sqrt{\gamma 1}\,(p\,\delta-\nu c0)\right)+\frac{1}{2\,\gamma 1}\,e^{(-w-\alpha)\sqrt{\gamma 1}}\,\eta 3\left(2\,e^{(w+\alpha)\sqrt{\gamma 1}}\left(-1+e^{(w+\alpha)\sqrt{\gamma 1}}\right)\gamma 0\sqrt{\gamma 1}+2\,e^{2(w+\alpha)\sqrt{\gamma 1}}\,\gamma 1\,(p\,\delta-\nu c0)\right)\right)\Big/\left(\sqrt{2}\sqrt{\left(w+\frac{1}{\gamma 1}(w+\alpha)\,\eta 3^2\,\sigma c^2\,\mathrm{Sinh}\left[(w+\alpha)\sqrt{\gamma 1}\right]^2\right)}\right)\right)$$

Equation [5.3] is the expression for the hazard rate for current smokers. In the operation to follow, the component expressions, equations 5.2a-e, developed directly above, as well as the estimated parameters for never-smokers, are substituted into the hazard rate expression,

[5.3]

$$h_c = D\left[\text{CDF}\left[\text{NormalDistribution}[0, \sigma\text{tox}_c[w+\alpha]], \left(\eta1\, w + \eta2\, w^2 + \eta3\, \text{tox}_c[w+\alpha]\right)\right], w\right] \Big/ \left(1 - \text{CDF}\left[\text{NormalDistribution}[0, \sigma\text{tox}_c[w+\alpha]], \left(\eta1\, w + \eta2\, w^2 + \eta3\, \text{tox}_c[w+\alpha]\right)\right]\right) \text{ /. }$$

$$\left\{\text{tox}_c[w+\alpha] \to \frac{1}{2\,\gamma1}\left(e^{-(w+\alpha)\sqrt{\gamma1}}\left(\left(-1+e^{(w+\alpha)\sqrt{\gamma1}}\right)^2 \gamma0 + \left(-1+e^{2(w+\alpha)\sqrt{\gamma1}}\right)\sqrt{\gamma1}\,(p\,\delta - \nu c0)\right)\right), \text{tox}_c'[w+\alpha] \to -\frac{1}{2\sqrt{\gamma1}}\left(e^{-(w+\alpha)\sqrt{\gamma1}}\left(\left(-1+e^{(w+\alpha)\sqrt{\gamma1}}\right)^2 \gamma0 + \left(-1+e^{2(w+\alpha)\sqrt{\gamma1}}\right)\sqrt{\gamma1}\,(p\,\delta - \nu c0)\right)\right) + \frac{1}{2\,\gamma1}\left(e^{-(w+\alpha)\sqrt{\gamma1}}\left(2\,e^{(w+\alpha)\sqrt{\gamma1}}\left(-1+e^{(w+\alpha)\sqrt{\gamma1}}\right)\gamma0\sqrt{\gamma1} + 2\,e^{2(w+\alpha)\sqrt{\gamma1}}\,\gamma1\,(p\,\delta - \nu c0)\right)\right),$$

$$\sigma\text{tox}_c[w+\alpha] \to \sqrt{w + \frac{(w+\alpha)\,\eta3^2\,\sigma c^2\,\text{Sinh}\left[(w+\alpha)\sqrt{\gamma1}\right]^2}{\gamma1}},$$

$$\sigma\text{tox}_c'[w+\alpha] \to \frac{1 + \frac{2\,(w+\alpha)\,\eta3^2\,\sigma c^2\,\text{Cosh}\left[(w+\alpha)\sqrt{\gamma1}\right]\text{Sinh}\left[(w+\alpha)\sqrt{\gamma1}\right]}{\sqrt{\gamma1}} + \frac{\eta3^2\,\sigma c^2\,\text{Sinh}\left[(w+\alpha)\sqrt{\gamma1}\right]^2}{\gamma1}}{2\sqrt{w + \frac{(w+\alpha)\,\eta3^2\,\sigma c^2\,\text{Sinh}\left[(w+\alpha)\sqrt{\gamma1}\right]^2}{\gamma1}}}\right\} \text{ /. }$$

$$\{\delta \to 1, \eta1 \to -1.5681, \eta2 \to 0.2205\}$$

$$
\begin{aligned}
&\Bigg(e^{-Y}\Bigg(-\Bigg(\Bigg(-1.5681\text{`}\,w+0.2205\text{`}\,w^{2}+\frac{1}{2\,\gamma 1}\\
&\quad e^{(-w-\alpha)\sqrt{\gamma 1}}\,\eta 3\,\left(\left(-1+e^{(w+\alpha)\sqrt{\gamma 1}}\right)^{2}\gamma 0+\left(-1+e^{2\,(w+\alpha)\sqrt{\gamma 1}}\right)\sqrt{\gamma 1}\,(p-\nu c0)\right)\Bigg)\\
&\quad\Bigg(1+\frac{2\,(w+\alpha)\,\eta 3^{2}\,\sigma c^{2}\,\mathrm{Cosh}\left[(w+\alpha)\sqrt{\gamma 1}\right]\mathrm{Sinh}\left[(w+\alpha)\sqrt{\gamma 1}\right]}{\sqrt{\gamma 1}}+\\
&\quad\frac{\eta 3^{2}\,\sigma c^{2}\,\mathrm{Sinh}\left[(w+\alpha)\sqrt{\gamma 1}\right]^{2}}{\gamma 1}\Bigg)\Bigg)\Bigg/\left(2\sqrt{2}\left(w+\frac{(w+\alpha)\,\eta 3^{2}\,\sigma c^{2}\,\mathrm{Sinh}\left[(w+\alpha)\sqrt{\gamma 1}\right]^{2}}{\gamma 1}\right)^{3/2}\right)+\\
&\quad\Bigg(-1.5681\text{`}+0.441\text{`}\,w-\frac{e^{(-w-\alpha)\sqrt{\gamma 1}}\,\eta 3\left(\left(-1+e^{(w+\alpha)\sqrt{\gamma 1}}\right)^{2}\gamma 0+\left(-1+e^{2\,(w+\alpha)\sqrt{\gamma 1}}\right)\sqrt{\gamma 1}\,(p-\nu c0)\right)}{2\sqrt{\gamma 1}}+\\
&\quad\frac{1}{2\,\gamma 1}e^{(-w-\alpha)\sqrt{\gamma 1}}\,\eta 3\left(2\,e^{(w+\alpha)\sqrt{\gamma 1}}\left(-1+e^{(w+\alpha)\sqrt{\gamma 1}}\right)\gamma 0\sqrt{\gamma 1}+2\,e^{2\,(w+\alpha)\sqrt{\gamma 1}}\,\gamma 1\,(p-\nu c0)\right)\Bigg)\Bigg/\\
&\quad\left(\sqrt{2}\sqrt{w+\frac{(w+\alpha)\,\eta 3^{2}\,\sigma c^{2}\,\mathrm{Sinh}\left[(w+\alpha)\sqrt{\gamma 1}\right]^{2}}{\gamma 1}}\right)\Bigg)\Bigg)\Bigg/\\
&\Bigg(\sqrt{\pi}\,\Bigg(1+\frac{1}{2}\Bigg(-1-\mathrm{Erf}\Bigg[\Bigg(-1.5681\text{`}\,w+0.2205\text{`}\,w^{2}+\frac{1}{2\,\gamma 1}e^{(-w-\alpha)\sqrt{\gamma 1}}\,\eta 3\\
&\quad\left(\left(-1+e^{(w+\alpha)\sqrt{\gamma 1}}\right)^{2}\gamma 0+\left(-1+e^{2\,(w+\alpha)\sqrt{\gamma 1}}\right)\sqrt{\gamma 1}\,(p-\nu c0)\right)\Bigg)\Bigg/\\
&\quad\left(\sqrt{2}\sqrt{w+\frac{(w+\alpha)\,\eta 3^{2}\,\sigma c^{2}\,\mathrm{Sinh}\left[(w+\alpha)\sqrt{\gamma 1}\right]^{2}}{\gamma 1}}\right)\Bigg]\Bigg)\Bigg)\Bigg)
\end{aligned}
$$

where : Y =

$$\left(-1.5681` \, w + 0.2205` \, w^2 + \frac{1}{2\,\gamma 1} e^{(-w-\alpha)\sqrt{\gamma 1}} \eta 3 \left(\left(-1 + e^{(w+\alpha)\sqrt{\gamma 1}}\right)^2 \gamma 0 + \left(-1 + e^{2\,(w+\alpha)\sqrt{\gamma 1}}\right)\sqrt{\gamma 1}\,(p - \nu c0)\right)\right)^2 \Big/ \left(2\left(w + \frac{(w+\alpha)\,\eta 3^2\,\sigma c^2\,\mathrm{Sinh}\left[(w+\alpha)\sqrt{\gamma 1}\right]^2}{\gamma 1}\right)\right)$$

The survival function for a current-smoke at age "ulage" (which is the upper limit of the integration over age), is the probability that the current-smoker will live beyond ulage (upper-limit of the integration over age), given that the respondent was alive at the base-line--his age in 1966, llagein66 (lower - limit of the integration over age--the respondent's age in 1966). The survival function is equal to the exponential of the integral of the negative of the hazard - rate over the period of observation, from llage to the age of the respondent when observation is complete, which is either: (1) the respondent's age when follow-up is completed (his age in 1999); or (2) his age at death. The Mathematica expression for the survival function is given by equation [5.4]. The Hold[] function tells Mathematica not to evaluate the expression. It will be evaluated when the individuals age, w, and his adjustment for when he started smoking, α, are substituted in.

[5.4]

$$g_c = \mathrm{Exp}\Big[\mathrm{Hold}\Big[\mathrm{NIntegrate}\Big[-\left(e^{-Y}\left(-\left(\left(-1.5681` \, w + 0.2205` \, w^2 + \frac{1}{2\,\gamma 1} e^{(-w-\alpha)\sqrt{\gamma 1}} \eta 3 \left(\left(-1 + e^{(w+\alpha)\sqrt{\gamma 1}}\right)^2 \gamma 0 + \left(-1 + e^{2\,(w+\alpha)\sqrt{\gamma 1}}\right)\sqrt{\gamma 1}\,(p - \nu c0)\right)\right)\right.\right.\right.$$

$$\left(1+\frac{1}{\sqrt{\gamma 1}}\,2\,(w+\alpha)\,\eta 3^{2}\,\sigma c^{2}\,\mathrm{Cosh}\left[(w+\alpha)\,\sqrt{\gamma 1}\right]\right.$$

$$\left.\left.\mathrm{Sinh}\left[(w+\alpha)\,\sqrt{\gamma 1}\right]+\frac{\eta 3^{2}\,\sigma c^{2}\,\mathrm{Sinh}\left[(w+\alpha)\,\sqrt{\gamma 1}\right]^{2}}{\gamma 1}\right)\right)\Big/$$

$$\left(2\,\sqrt{2}\,\left(w+\frac{(w+\alpha)\,\eta 3^{2}\,\sigma c^{2}\,\mathrm{Sinh}\left[(w+\alpha)\,\sqrt{\gamma 1}\right]^{2}}{\gamma 1}\right)^{3/2}\right)+$$

$$\left(-1.5681\grave{}+0.441\grave{}\,w-\frac{1}{2\,\sqrt{\gamma 1}}\,e^{(-w-\alpha)\,\sqrt{\gamma 1}}\,\eta 3\right.$$

$$\left(\left(-1+e^{(w+\alpha)\,\sqrt{\gamma 1}}\right)^{2}\gamma 0+\left(-1+e^{2\,(w+\alpha)\,\sqrt{\gamma 1}}\right)\sqrt{\gamma 1}\,(p-\nu c0)\right)+$$

$$\frac{1}{2\,\gamma 1}\,e^{(-w-\alpha)\,\sqrt{\gamma 1}}\,\eta 3\,\left(2\,e^{(w+\alpha)\,\sqrt{\gamma 1}}\,\left(-1+e^{(w+\alpha)\,\sqrt{\gamma 1}}\right)\right.$$

$$\left.\left.\gamma 0\,\sqrt{\gamma 1}+2\,e^{2\,(w+\alpha)\,\sqrt{\gamma 1}}\,\gamma 1\,(p-\nu c0)\right)\right)\Big/$$

$$\left.\left.\left(\sqrt{2}\,\sqrt{w+\frac{(w+\alpha)\,\eta 3^{2}\,\sigma c^{2}\,\mathrm{Sinh}\left[(w+\alpha)\,\sqrt{\gamma 1}\right]^{2}}{\gamma 1}}\right)\right)\right)\Big/$$

$$\left(\sqrt{\pi}\,\left(1+\frac{1}{2}\,\left(-1-\mathrm{Erf}\left[\left(-1.5681\grave{}\,w+0.2205\grave{}\,w^{2}+\frac{1}{2\,\gamma 1}\right.\right.\right.\right.\right.$$

$$e^{(-w-\alpha)\sqrt{\gamma1}}\;\eta3\left(\left(-1+e^{(w+\alpha)\sqrt{\gamma1}}\right)^{2}\gamma0+\left(-1+e^{2(w+\alpha)\sqrt{\gamma1}}\right)\sqrt{\gamma1}\;(p-\nu c0)\right)\Bigg)\Bigg/$$

$$\left(\sqrt{2}\sqrt{w+\frac{(w+\alpha)\,\eta3^{2}\,\sigma c^{2}\,\mathrm{Sinh}\left[(w+\alpha)\sqrt{\gamma1}\right]^{2}}{\gamma1}}\;\right]\Bigg)\Bigg)\Bigg)\Bigg),$$

{w, llagein66, ulage}]]]

$$\mathrm{Hold}\Big[\mathrm{NIntegrate}\Big[-\frac{e^{-Y}\left(-\frac{\left(-1.5681\,w+0.2205\,w^{2}+\frac{e^{(-w-\alpha)\sqrt{\gamma1}}\,\eta3\left(\left(-1+e^{(w+\alpha)\sqrt{\gamma1}}\right)^{2}\gamma0+\left(-1+e^{2(w+\alpha)\sqrt{\gamma1}}\right)\sqrt{\gamma1}\,(p-\nu c0)\right)}{2\,\gamma1}\right)\left(1+\right.}{2\sqrt{2}\left(w+\frac{(w+\alpha)\,\eta3^{2}\,\sigma c^{2}\,\mathrm{Sinh}[(w+\alpha)}{\gamma1}\right.}\right.}{}$$

e

g_c =
Exp[Hold [NIntegrate [-(Exp[-Y] ((-A B) / C + (D / E))) / F, {w, llagein66 , ulage}]]

$$A=\left(-1.5681`\,w+0.2205`\,w^{2}+\frac{1}{2\,\gamma1}\,e^{(-w-\alpha)\sqrt{\gamma1}}\;\eta3\left(\left(-1+e^{(w+\alpha)\sqrt{\gamma1}}\right)^{2}\gamma0+\left(-1+e^{2(w+\alpha)\sqrt{\gamma1}}\right)\sqrt{\gamma1}\;(p-\nu c0)\right)\right)$$

$$B=\left(1+\frac{2\,(w+\alpha)\,\sigma c^{2}\,\mathrm{Cosh}\left[(w+\alpha)\sqrt{\gamma1}\right]\mathrm{Sinh}\left[(w+\alpha)\sqrt{\gamma1}\right]}{\sqrt{\gamma1}}+\frac{\sigma c^{2}\,\mathrm{Sinh}\left[(w+\alpha)\sqrt{\gamma1}\right]^{2}}{\gamma1}\right)\Bigg)$$

$$C=\left(2\sqrt{2}\left(w+\frac{(w+\alpha)\,\sigma c^{2}\,\mathrm{Sinh}\left[(w+\alpha)\sqrt{\gamma1}\right]^{2}}{\gamma1}\right)^{3/2}\right)$$

$$D=-1.5681`+0.441`\,w+\eta3\left(-\frac{1}{2\sqrt{\gamma1}}e^{(-w-\alpha)\sqrt{\gamma1}}\left(\left(-1+e^{(w+\alpha)\sqrt{\gamma1}}\right)^{2}\gamma0+\left(-1+e^{2(w+\alpha)\sqrt{\gamma1}}\right)\sqrt{\gamma1}\;(p-\nu c0)\right)+\frac{1}{2\,\gamma1}\,e^{(-w-\alpha)\sqrt{\gamma1}}\left(2\,e^{(w+\alpha)\sqrt{\gamma1}}\left(-1+e^{(w+\alpha)\sqrt{\gamma1}}\right)\gamma0\,\sqrt{\gamma1}+2\,e^{2(w+\alpha)\sqrt{\gamma1}}\,\gamma1\,(p-\nu c0)\right)\right.$$

$$E = \sqrt{2}\,\sqrt{w + \frac{(w+\alpha)\,\sigma c^2\,\mathrm{Sinh}\left[(w+\alpha)\,\sqrt{\gamma 1}\right]^2}{\gamma 1}}$$

$$F = \sqrt{\pi}\left(1 + \frac{1}{2}\left(-1 - \mathrm{Erf}\left[\left(-1.5681`\,w + 0.2205`\,w^2 + \frac{1}{2\,\gamma 1}\,e^{(-w-\alpha)\sqrt{\gamma 1}}\,\eta 3\left(\left(-1 + e^{(w+\alpha)\sqrt{\gamma 1}}\right)^2 \gamma 0 + \left(-1 + e^{2(w+\alpha)\sqrt{\gamma 1}}\right)\sqrt{\gamma 1}\,(p - \nu c0)\right)\right) \Big/ \left(\sqrt{2}\,\sqrt{w + \frac{(w+\alpha)\,\sigma c^2\,\mathrm{Sinh}\left[(w+\alpha)\,\sqrt{\gamma 1}\right]^2}{\gamma 1}}\right)\right]\right)\right)$$

To construct the probability density function of the life-span, the probability of living until time ulage and then dying at time ulage, we need the hazard-rate evaluated at time of death. This value is given by equation [5.5],

[5.5]

$ht_c = h_c$ /. $w \to$ ulage

$$\Bigg(e^{-Y}\Bigg(-\Bigg(\Bigg(-1.5681\grave{}\ \text{ulage}+0.2205\grave{}\ \text{ulage}^2+$$

$$\frac{1}{2\,\gamma1}\,e^{(-\text{ulage}-\alpha)\sqrt{\gamma1}}\,\eta3\,\Big(\big(-1+e^{(\text{ulage}+\alpha)\sqrt{\gamma1}}\big)^2\,\gamma0+\big(-1+e^{2\,(\text{ulage}+\alpha)\sqrt{\gamma1}}\big)\sqrt{\gamma1}\,(\text{p}-\nu\text{c0})\Big)\Bigg)$$

$$\Bigg(1+\frac{1}{\sqrt{\gamma1}}2\,(\text{ulage}+\alpha)\,\eta3^2\,\sigma\text{c}^2\,\text{Cosh}\Big[(\text{ulage}+\alpha)\sqrt{\gamma1}\Big]\,\text{Sinh}\Big[(\text{ulage}+\alpha)\sqrt{\gamma1}\Big]+$$

$$\frac{\eta3^2\,\sigma\text{c}^2\,\text{Sinh}\Big[(\text{ulage}+\alpha)\sqrt{\gamma1}\Big]^2}{\gamma1}\Bigg)\Bigg)\Bigg/$$

$$\left(2\sqrt{2}\left(\text{ulage}+\frac{(\text{ulage}+\alpha)\,\eta3^2\,\sigma\text{c}^2\,\text{Sinh}\Big[(\text{ulage}+\alpha)\sqrt{\gamma1}\Big]^2}{\gamma1}\right)^{3/2}\right)+$$

$$\Bigg(-1.5681\grave{}+0.441\grave{}\ \text{ulage}-\frac{1}{2\sqrt{\gamma1}}e^{(-\text{ulage}-\alpha)\sqrt{\gamma1}}\,\eta3$$

$$\Big(\big(-1+e^{(\text{ulage}+\alpha)\sqrt{\gamma1}}\big)^2\,\gamma0+\big(-1+e^{2\,(\text{ulage}+\alpha)\sqrt{\gamma1}}\big)\sqrt{\gamma1}\,(\text{p}-\nu\text{c0})\Big)+\frac{1}{2\,\gamma1}\,e^{(-\text{ulage}-\alpha)\sqrt{\gamma1}}$$

$$\eta3\,\Big(2\,e^{(\text{ulage}+\alpha)\sqrt{\gamma1}}\,\big(-1+e^{(\text{ulage}+\alpha)\sqrt{\gamma1}}\big)\,\gamma0\,\sqrt{\gamma1}+2\,e^{2\,(\text{ulage}+\alpha)\sqrt{\gamma1}}\,\gamma1\,(\text{p}-\nu\text{c0})\Big)\Bigg)\Bigg/$$

$$\left(\sqrt{2}\,\sqrt{\text{ulage}+\frac{(\text{ulage}+\alpha)\,\eta3^2\,\sigma\text{c}^2\,\text{Sinh}\Big[(\text{ulage}+\alpha)\sqrt{\gamma1}\Big]^2}{\gamma1}}\right)\Bigg)\Bigg)\Bigg/$$

$$\Bigg(\sqrt{\pi}\,\Bigg(1+\frac{1}{2}\Bigg(-1-\text{Erf}\Bigg[\Bigg(-1.5681\grave{}\ \text{ulage}+0.2205\grave{}\ \text{ulage}^2+$$

$$\frac{1}{2\,\gamma1}\,e^{(-\text{ulage}-\alpha)\sqrt{\gamma1}}\,\eta3\,\Big(\big(-1+e^{(\text{ulage}+\alpha)\sqrt{\gamma1}}\big)^2\,\gamma0+\big(-1+e^{2\,(\text{ulage}+\alpha)\sqrt{\gamma1}}\big)\sqrt{\gamma1}\,(\text{p}-\nu\text{c0})\Big)\Bigg)\Bigg/$$

$$\left(\sqrt{2}\,\sqrt{\text{ulage}+\frac{(\text{ulage}+\alpha)\,\eta3^2\,\sigma\text{c}^2\,\text{Sinh}\Big[(\text{ulage}+\alpha)\sqrt{\gamma1}\Big]^2}{\gamma1}}\right)\Bigg]\Bigg)\Bigg)\Bigg)$$

The probability density function of the random life span for a current smoker is the product of the hazard-rate evaluated at time of death and the survival function evaluated to the time of death, equation [5.6],

[5.6]

$$\mathbf{f_c = ht_c\, g_c}$$

f_c can be obtained by substitution.

■ Section 6: The dynamic Normal survival model for Former-Smokers.

In the analysis of the survival of former smokers, as with the other smoking statuses all time is measured in decades. te is the time the individual smoked, u is the time he abstained from smoking. w is his age, past 17 and α is the time adjustment to convert age into time smoked. Moreover, te + u = w + α. The distribution of tobacco-exposure of former-smokers at age w, (u + te - α) in decades beyound age 17, is Normal. Its expected value and variance are given by equation [6.1], (see Appendix 1 for derivation). Note that both the expected value and the variance are composed of two terms. The first term (in both) is the expected tobacco-exposure and variance that occured while the former-smoker was a current smoker. The second term is the "contribution" of the former-smoker's abstention to his tobacco exposure and its variance,

[6.1]

$$\text{tox}_f[u \mid te] \sim N\left[\left(\left(\frac{e^{-te\sqrt{\gamma 1}}\left(\left(-1+e^{te\sqrt{\gamma 1}}\right)^2 \gamma_0+\left(-1+e^{2\,te\sqrt{\gamma 1}}\right)\sqrt{\gamma 1}\,(p\,\delta-\nu_{c0})\right)}{2\,\gamma 1}\right)+\left(\frac{1}{2\,\gamma 1}e^{-(te+u)\sqrt{\gamma 1}}\left(-1+e^{u\sqrt{\gamma 1}}\right)\left(\left(-1+e^{(2\,te+u)\sqrt{\gamma 1}}\right)\gamma_0+\sqrt{\gamma 1}\left(\left(-1+e^{te\sqrt{\gamma 1}}\right)\left(-1+e^{(te+u)\sqrt{\gamma 1}}\right)p\,\delta-\left(1+e^{(2\,te+u)\sqrt{\gamma 1}}\right)\nu_{c0}\right)\right)\right)\right),\left(\left(\frac{1}{\gamma 1}\left(\frac{1}{4}te\left(-2+\text{Cosh}\left[2\,(te-u)\sqrt{\gamma 1}\right]+\text{Cosh}\left[2\,(te+u)\sqrt{\gamma 1}\right]\right)\sigma_c^2\right)+u\,\text{Sinh}\left[u\sqrt{\gamma 1}\right]^2\sigma_f^2\right)\right)\right]$$

In addition to the age and smoking related terms discussed immediately above,

$\gamma 0$ is the trend in the time rate of change of the purge rate;

$\gamma 1$ is the marginal effect of a unit of tobacco-toxin on the time rate of change of the purge rate;

$\nu c0$ is the purge rate when smoking is first initiated,

p is the packs of cigarettes smoked per day;

δ is the toxins per pack smoked, and

σc^2 & σf^2 are, respectively, the square of the standard deviation of the Brownian motion process of the random variable in the specification of the time rate of change of the purge-rate for current and former smokers. This Brownian motion process has been "standardized" by the standard deviation of the Brownian motion process of describing the vicitudes of life, which is the dynamic process leading to the propensity to die by time w for a never-smoker, and applies to all respondents.

The propensity for a former-smoker to be dead by age w (in decades), given he smoked for te decades is specified by equation [6.2],

[6.2] $\text{death*}[u|te,\alpha] = \eta 1\ (u+te-\alpha) + \eta 2\ (u+te-\alpha)^2 + \eta 3\ E[tox_c[te]]$

$+ \eta 4\ E[tox_f[u\mid te]] + \varsigma[u|\alpha,te],$

where:

E[] is the expectation operator;

$\varsigma[u|\alpha,te]$ is a random variable with a Normal distribution whose expected value equals zero; and whose variance at age w, for a former-smoker who smoked a duration te decades equals the quantity (See Appendix 1),

$$V[\varsigma[u\mid\alpha,te]] = (u+te-\alpha) + \eta 3^2 \left(\frac{1}{\gamma 1}\left(\frac{1}{4}\,te\left(-2+\text{Cosh}\left[2\,(te-u)\sqrt{\gamma 1}\right]+\text{Cosh}\left[2\,(te+u)\sqrt{\gamma 1}\right]\right)\right)\right)\sigma_c^2 + \eta 4^2\left(u\,\text{Sinh}\left[u\sqrt{\gamma 1}\right]^2\sigma_f^2\right)$$

As in Section 5 above, we now detail expressions for the components of the hazard rate. From equation [6.1] the expected value of the former-smokers tobacco exposure is given by equation [6.3],

[6.3a]

$$\mathbf{tox_f} = \left(\frac{1}{2\gamma 1} e^{-te\sqrt{\gamma 1}}\left(\left(-1+e^{te\sqrt{\gamma 1}}\right)^2\gamma_0+\left(-1+e^{2\,te\sqrt{\gamma 1}}\right)\sqrt{\gamma 1}\,(p\,\delta-\nu_{c0})\right)\right) + \left(\frac{1}{2\gamma 1} e^{-(te+u)\sqrt{\gamma 1}}\left(-1+e^{u\sqrt{\gamma 1}}\right)\left(\left(-1+e^{(2\,te+u)\sqrt{\gamma 1}}\right)\gamma_0+\sqrt{\gamma 1}\left(\left(-1+e^{te\sqrt{\gamma 1}}\right)\left(-1+e^{(te+u)\sqrt{\gamma 1}}\right)p\,\delta-\left(1+e^{(2\,te+u)\sqrt{\gamma 1}}\right)\nu_{c0}\right)\right)\right)$$

$$\frac{e^{-te\sqrt{\gamma 1}}\left(\left(-1+e^{te\sqrt{\gamma 1}}\right)^2\gamma_0+\left(-1+e^{2\,te\sqrt{\gamma 1}}\right)\sqrt{\gamma 1}\,(p\,\delta-\nu_{c0})\right)}{2\gamma 1} + \frac{1}{2\gamma 1} e^{(-te-u)\sqrt{\gamma 1}}\left(-1+e^{u\sqrt{\gamma 1}}\right)\left(\left(-1+e^{(2\,te+u)\sqrt{\gamma 1}}\right)\gamma_0+\sqrt{\gamma 1}\left(\left(-1+e^{te\sqrt{\gamma 1}}\right)\left(-1+e^{(te+u)\sqrt{\gamma 1}}\right)p\,\delta-\left(1+e^{(2\,te+u)\sqrt{\gamma 1}}\right)\nu_{c0}\right)\right)$$

Equation [6.3b] is the expression for the time rate of change of the expected value of tox_f [u|te],
[6.3b]

$tox_f' = D[tox_f, u]$

$$\frac{1}{2\sqrt{\gamma1}} e^{(-te-u)\sqrt{\gamma1}+u\sqrt{\gamma1}} \left(\left(-1+e^{(2\,te+u)\sqrt{\gamma1}}\right)\gamma_0 + \sqrt{\gamma1}\left(\left(-1+e^{te\sqrt{\gamma1}}\right)\left(-1+e^{(te+u)\sqrt{\gamma1}}\right)p\,\delta - \left(1+e^{(2\,te+u)\sqrt{\gamma1}}\right)\nu_{c0}\right)\right) - \frac{1}{2\sqrt{\gamma1}} e^{(-te-u)\sqrt{\gamma1}}\left(-1+e^{u\sqrt{\gamma1}}\right)\left(\left(-1+e^{(2\,te+u)\sqrt{\gamma1}}\right)\gamma_0 + \sqrt{\gamma1}\left(\left(-1+e^{te\sqrt{\gamma1}}\right)\left(-1+e^{(te+u)\sqrt{\gamma1}}\right)p\,\delta - \left(1+e^{(2\,te+u)\sqrt{\gamma1}}\right)\nu_{c0}\right)\right) + \frac{1}{2\gamma1} e^{(-te-u)\sqrt{\gamma1}}\left(-1+e^{u\sqrt{\gamma1}}\right)\left(e^{(2\,te+u)\sqrt{\gamma1}}\sqrt{\gamma1}\,\gamma_0 + \sqrt{\gamma1}\left(e^{(te+u)\sqrt{\gamma1}}\left(-1+e^{te\sqrt{\gamma1}}\right)p\sqrt{\gamma1}\,\delta - e^{(2\,te+u)\sqrt{\gamma1}}\sqrt{\gamma1}\,\nu_{c0}\right)\right)$$

From the remarks for equation [6.2], equation [6.3c] is the expression for the standard deviation of the error term in the latent index of death, ς[w, α | te],denoted $\sigma tox_f[w+\alpha]$,

[6.3c]

$$\boldsymbol{\sigma\, tox_f = Sqrt\left[(u+te-\alpha) + \eta3^2\left(\frac{1}{\gamma1}\left(\frac{1}{4}te\left(-2+Cosh\left[2\,(te-u)\sqrt{\gamma1}\right] + Cosh\left[2\,(te+u)\sqrt{\gamma1}\right]\right)\sigma_c^2\right)\right) + \eta4^2\left(u\,Sinh\left[u\sqrt{\gamma1}\right]^2\sigma_f^2\right)\right]}$$

$$\sqrt{\left(te+u-\alpha+\frac{te\,\eta3^2\left(-2+Cosh\left[2\,(te-u)\sqrt{\gamma1}\right]+Cosh\left[2\,(te+u)\sqrt{\gamma1}\right]\right)\sigma_c^2}{4\,\gamma1}+u\,\eta4^2\,Sinh\left[u\sqrt{\gamma1}\right]^2\sigma_f^2\right)}$$

Equation [6.3d] is the expression for the time rate of change in the standard deviation of the error term in the latent index of death, denoted $\sigma tox_f'$,

[6.3d]

$$\sigma tox_f{}' = D\left[\sqrt{\left((te+u-\alpha) + \frac{te\,\eta 3^2\left(-2+\text{Cosh}\left[2\,(te-u)\sqrt{\gamma 1}\right]+\text{Cosh}\left[2\,(te+u)\sqrt{\gamma 1}\right]\right)\sigma_c^2}{4\,\gamma 1} + u\,\eta 4^2\,\text{Sinh}\left[u\sqrt{\gamma 1}\right]^2\sigma_f^2\right)},\,u\right]$$

$$\left(1+\frac{1}{4\,\gamma 1}te\,\eta 3^2\left(-2\sqrt{\gamma 1}\,\text{Sinh}\left[2\,(te-u)\sqrt{\gamma 1}\right]+2\sqrt{\gamma 1}\,\text{Sinh}\left[2\,(te+u)\sqrt{\gamma 1}\right]\right)\sigma_c^2 + 2\,u\sqrt{\gamma 1}\,\eta 4^2\,\text{Cosh}\left[u\sqrt{\gamma 1}\right]\text{Sinh}\left[u\sqrt{\gamma 1}\right]\sigma_f^2+\eta 4^2\,\text{Sinh}\left[u\sqrt{\gamma 1}\right]^2\sigma_f^2\right) \Big/ \left(2\sqrt{\left(te+u-\alpha+\frac{te\,\eta 3^2\left(-2+\text{Cosh}\left[2\,(te-u)\sqrt{\gamma 1}\right]+\text{Cosh}\left[2\,(te+u)\sqrt{\gamma 1}\right]\right)\sigma_c^2}{4\,\gamma 1}+u\,\eta 4^2\,\text{Sinh}\left[u\sqrt{\gamma 1}\right]^2\sigma_f^2\right)}\right)$$

Equation [6.3e] is the expression for the time rate of change in the probability of dying,

[6.3e] $\partial\text{Pr[death]}/\partial u =$

$$D\Big[\text{CDF}\Big[\text{NormalDistribution}\Big[0,\ \text{Sqrt}\Big[(u+te-\alpha)+\eta 3^2\left(\frac{1}{\gamma 1}\left(\frac{1}{4}te\left(-2+\text{Cosh}\left[2\,(te-u)\sqrt{\gamma 1}\right]+\text{Cosh}\left[2\,(te+u)\sqrt{\gamma 1}\right]\right)\right)\right)\sigma_c^2+\eta 4^2\left(u\,\text{Sinh}\left[u\sqrt{\gamma 1}\right]^2\sigma_f^2\right)\Big]\Big],$$

$$\Big(\eta 1\,(te+u-\alpha)+\eta 2\,(te+u-\alpha)^2+ \eta 3\left(\frac{e^{-te\sqrt{\gamma 1}}\left(\left(-1+e^{te\sqrt{\gamma 1}}\right)^2\gamma_0+\left(-1+e^{2\,te\sqrt{\gamma 1}}\right)\sqrt{\gamma 1}\,(p\,\delta-\nu_{c0})\right)}{2\,\gamma 1}\right)+$$

$$\eta 4\left(\frac{1}{2\,\gamma 1}e^{-(te+u)\sqrt{\gamma 1}}\left(-1+e^{u\sqrt{\gamma 1}}\right)\left(\left(-1+e^{(2\,te+u)\sqrt{\gamma 1}}\right)\gamma_0+\sqrt{\gamma 1}\left(\left(-1+e^{te\sqrt{\gamma 1}}\right)\left(-1+e^{(te+u)\sqrt{\gamma 1}}\right)p\,\delta-\left(1+e^{(2\,te+u)\sqrt{\gamma 1}}\right)\nu_{c0}\right)\right)\right)\Big)\Big],\,u\Big]$$

$$= \frac{1}{\sqrt{\pi}} e^{-(A/B)}$$

$$\left(-\left(\left((te+u-\alpha)\,\eta 1 + (te+u-\alpha)^2\,\eta 2 + \frac{1}{2\,\gamma 1} e^{-te\sqrt{\gamma 1}}\,\eta 3 \left(\left(-1+e^{te\sqrt{\gamma 1}}\right)^2 \gamma_0 + \left(-1+e^{2\,te\sqrt{\gamma 1}}\right)\sqrt{\gamma 1}\right.\right.\right.\right.$$

$$\left.(p\,\delta - \nu_{c0})\right) + \frac{1}{2\,\gamma 1} e^{(-te-u)\sqrt{\gamma 1}} \left(-1+e^{u\sqrt{\gamma 1}}\right)\eta 4 \left(\left(-1+e^{(2\,te+u)\sqrt{\gamma 1}}\right)\gamma_0 + \right.$$

$$\left.\left.\sqrt{\gamma 1}\left(\left(-1+e^{te\sqrt{\gamma 1}}\right)\left(-1+e^{(te+u)\sqrt{\gamma 1}}\right) p\,\delta - \left(1+e^{(2\,te+u)\sqrt{\gamma 1}}\right)\nu_{c0}\right)\right)\right)$$

$$\left(1 + \frac{1}{4\,\gamma 1}\,te\,\eta 3^2 \left(-2\sqrt{\gamma 1}\,\mathrm{Sinh}\left[2\,(te-u)\sqrt{\gamma 1}\right] + 2\sqrt{\gamma 1}\,\mathrm{Sinh}\left[2\,(te+u)\sqrt{\gamma 1}\right]\right)\sigma_c^2 + \right.$$

$$\left.\left.\left. 2\,u\sqrt{\gamma 1}\,\eta 4^2\,\mathrm{Cosh}\left[u\sqrt{\gamma 1}\right]\mathrm{Sinh}\left[u\sqrt{\gamma 1}\right]\sigma_f^2 + \eta 4^2\,\mathrm{Sinh}\left[u\sqrt{\gamma 1}\right]^2\sigma_f^2\right)\right) \right/$$

$$\left(2\sqrt{2}\left(te+u-\alpha+\frac{1}{4\,\gamma 1}\,te\,\eta 3^2\left(-2+\mathrm{Cosh}\left[2\,(te-u)\sqrt{\gamma 1}\right]+\mathrm{Cosh}\left[2\,(te+u)\sqrt{\gamma 1}\right]\right)\sigma_c^2 + \right.\right.$$

$$\left.\left. u\,\eta 4^2\,\mathrm{Sinh}\left[u\sqrt{\gamma 1}\right]^2\sigma_f^2\right)^{3/2}\right) +$$

$$\left(\eta 1 + 2\,(te+u-\alpha)\,\eta 2 + \frac{1}{2\sqrt{\gamma 1}} e^{(-te-u)\sqrt{\gamma 1}+u\sqrt{\gamma 1}}\,\eta 4\left(\left(-1+e^{(2\,te+u)\sqrt{\gamma 1}}\right)\gamma_0 + \right.\right.$$

$$\left.\sqrt{\gamma 1}\left(\left(-1+e^{te\sqrt{\gamma 1}}\right)\left(-1+e^{(te+u)\sqrt{\gamma 1}}\right) p\,\delta - \left(1+e^{(2\,te+u)\sqrt{\gamma 1}}\right)\nu_{c0}\right)\right) -$$

$$\frac{1}{2\sqrt{\gamma 1}} e^{(-te-u)\sqrt{\gamma 1}}\left(-1+e^{u\sqrt{\gamma 1}}\right)\eta 4\left(\left(-1+e^{(2\,te+u)\sqrt{\gamma 1}}\right)\gamma_0 + \right.$$

$$\left.\sqrt{\gamma 1}\left(\left(-1+e^{te\sqrt{\gamma 1}}\right)\left(-1+e^{(te+u)\sqrt{\gamma 1}}\right) p\,\delta - \left(1+e^{(2\,te+u)\sqrt{\gamma 1}}\right)\nu_{c0}\right)\right) +$$

$$\frac{1}{2\,\gamma 1} e^{(-te-u)\sqrt{\gamma 1}}\left(-1+e^{u\sqrt{\gamma 1}}\right)\eta 4\left(e^{(2\,te+u)\sqrt{\gamma 1}}\sqrt{\gamma 1}\,\gamma_0 + \right.$$

$$\left.\left.\left.\sqrt{\gamma 1}\left(e^{(te+u)\sqrt{\gamma 1}}\left(-1+e^{te\sqrt{\gamma 1}}\right) p\sqrt{\gamma 1}\,\delta - e^{(2\,te+u)\sqrt{\gamma 1}}\sqrt{\gamma 1}\,\nu_{c0}\right)\right)\right)\right/$$

$$\left(\sqrt{2}\sqrt{\left(te+u-\alpha+\frac{1}{4\,\gamma 1}\,te\,\eta 3^2\left(-2+\mathrm{Cosh}\left[2\,(te-u)\sqrt{\gamma 1}\right]+\mathrm{Cosh}\left[2\,(te+u)\sqrt{\gamma 1}\right]\right)\sigma_c^2 + \right.}\right.$$

$$\left.\left.\left. u\,\eta 4^2\,\mathrm{Sinh}\left[u\sqrt{\gamma 1}\right]^2\sigma_f^2\right)\right)\right)$$

where (A / B) is defined as follows :

$$\left((te+u-\alpha)\,\eta1+(te+u-\alpha)^2\,\eta2+\frac{1}{2\,\gamma1}\,e^{-te\sqrt{\gamma1}}\,\eta3\,\left(\left(-1+e^{te\sqrt{\gamma1}}\right)^2\gamma_0+\left(-1+e^{2\,te\sqrt{\gamma1}}\right)\sqrt{\gamma1}\,(p\,\delta-\nu_{c0})\right)+\frac{1}{2\,\gamma1}\,e^{(-te-u)\sqrt{\gamma1}}\left(-1+e^{u\sqrt{\gamma1}}\right)\eta4\,\left(\left(-1+e^{(2\,te+u)\sqrt{\gamma1}}\right)\gamma_0+\sqrt{\gamma1}\,\left(\left(-1+e^{te\sqrt{\gamma1}}\right)\left(-1+e^{(te+u)\sqrt{\gamma1}}\right)p\,\delta-\left(1+e^{(2\,te+u)\sqrt{\gamma1}}\right)\nu_{c0}\right)\right)\right)^2\Big/$$

$$\left(2\left(te+u-\alpha+\frac{1}{4\,\gamma1}\,te\,\eta3^2\left(-2+\text{Cosh}\left[2\,(te-u)\sqrt{\gamma1}\right]+\text{Cosh}\left[2\,(te+u)\sqrt{\gamma1}\right]\right)\sigma_c^2+u\,\eta4^2\,\text{Sinh}\left[u\sqrt{\gamma1}\right]^2\sigma_f^2\right)\right)\ /.$$

$\{\delta\to 1,\ \eta1\to -1.5681,\ \eta2\to 0.2205,\ \eta3\to 0.1107,\ \gamma_0\to 0,$
$\gamma1\to \text{Exp}[-21.5052],$
$\nu_{c0}\to -1.7248,\ \sigma_c\to \text{Exp}[-27.1608]\}$

(A / B) =

$$\left(2587.691333548374`\,e^{-0.000021389722677666222`\,te}\left(-1+e^{0.000042779445355332445`\,te}\right)(1.7248`+p)-1.5681`\,(te+u-\alpha)+0.2205`\,(te+u-\alpha)^2+23375.71213684168`\,e^{0.000021389722677666222`\,(-te-u)}\left(-1+e^{0.000021389722677666222`\,u}\right)\left(1.7248`\left(1+e^{0.000021389722677666222`\,(2\,te+u)}\right)+\left(-1+e^{0.000021389722677666222`\,te}\right)\left(-1+e^{0.000021389722677666222`\,(te+u)}\right)p\right)\eta4\right)^2\Big/$$

$$\left(2\left(te+u-\alpha+1.714959287456654`*\text{^}-17\ te\,(-2+\text{Cosh}[0.000042779445355332445`\,(te-u)]+\text{Cosh}[0.000042779445355332445`\,(te+u)])+u\,\eta4^2\,\text{Sinh}[0.000021389722677666222`\,u]^2\,\sigma_f^2\right)\right)$$

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The hazard rate for former smokers, h_f, is the ratio of the time rate of change in the probability of dying, divided by the probability of living, equation [6.4],

[6.4]

$$h_f\ =\ \frac{1}{\sqrt{\pi}}\,e^{-A/B}$$

$$\left(-\left(\left((te+u-\alpha)\,\eta1+(te+u-\alpha)^2\,\eta2+\frac{1}{2\,\gamma1}e^{-te\sqrt{\gamma1}}\,\eta3\,\left(\left(-1+e^{te\sqrt{\gamma1}}\right)^2\gamma_0+\left(-1+e^{2\,te\sqrt{\gamma1}}\right)\sqrt{\gamma1}\,(p\,\delta-\nu_{c0})\right)+\frac{1}{2\,\gamma1}\,e^{(-te-u)\sqrt{\gamma1}}\left(-1+e^{u\sqrt{\gamma1}}\right)\eta4\,\left(\left(-1+e^{(2\,te+u)\sqrt{\gamma1}}\right)\right.\right.\right.$$

$$\gamma_0 + \sqrt{\gamma 1}\left(\left(-1 + e^{te\sqrt{\gamma 1}}\right)\left(-1 + e^{(te+u)\sqrt{\gamma 1}}\right) p\,\delta - \left(1 + e^{(2\,te+u)\sqrt{\gamma 1}}\right)\nu_{c0}\right)\right)\Big)$$

$$\left(\frac{1}{4\,\gamma 1}\,te\,\eta 3^2\left(-2\sqrt{\gamma 1}\,\text{Sinh}\left[2\,(te-u)\sqrt{\gamma 1}\right] + 2\sqrt{\gamma 1}\,\text{Sinh}\left[2\,(te+u)\sqrt{\gamma 1}\right]\right)\sigma_c^2 +\right.$$

$$\left.2\,u\sqrt{\gamma 1}\,\eta 4^2\,\text{Cosh}\left[u\sqrt{\gamma 1}\right]\text{Sinh}\left[u\sqrt{\gamma 1}\right]\sigma_f^2 + \eta 4^2\,\text{Sinh}\left[u\sqrt{\gamma 1}\right]^2\sigma_f^2\right)\Big)\Big/$$

$$\left(2\sqrt{2}\left((u + te - \alpha) + \frac{1}{4\,\gamma 1}te\,\eta 3^2\left(-2 + \text{Cosh}\left[2\,(te-u)\sqrt{\gamma 1}\right] +\right.\right.\right.$$

$$\left.\left.\text{Cosh}\left[2\,(te+u)\sqrt{\gamma 1}\right]\right)\sigma_c^2 + u\,\eta 4^2\,\text{Sinh}\left[u\sqrt{\gamma 1}\right]^2\sigma_f^2\right)^{3/2}\Big) +$$

$$\left(\eta 1 + 2\,(te + u - \alpha)\,\eta 2 + \frac{1}{2\sqrt{\gamma 1}}\,e^{(-te-u)\sqrt{\gamma 1}+u\sqrt{\gamma 1}}\,\eta 4\left(\left(-1 + e^{(2\,te+u)\sqrt{\gamma 1}}\right)\gamma_0 +\right.\right.$$

$$\left.\sqrt{\gamma 1}\left(\left(-1 + e^{te\sqrt{\gamma 1}}\right)\left(-1 + e^{(te+u)\sqrt{\gamma 1}}\right)p\,\delta - \left(1 + e^{(2\,te+u)\sqrt{\gamma 1}}\right)\nu_{c0}\right)\right) -$$

$$\frac{1}{2\sqrt{\gamma 1}}\,e^{(-te-u)\sqrt{\gamma 1}}\left(-1 + e^{u\sqrt{\gamma 1}}\right)\eta 4\left(\left(-1 + e^{(2\,te+u)\sqrt{\gamma 1}}\right)\gamma_0 +\right.$$

$$\left.\sqrt{\gamma 1}\left(\left(-1 + e^{te\sqrt{\gamma 1}}\right)\left(-1 + e^{(te+u)\sqrt{\gamma 1}}\right)p\,\delta - \left(1 + e^{(2\,te+u)\sqrt{\gamma 1}}\right)\nu_{c0}\right)\right) +$$

$$\frac{1}{2\,\gamma 1}\,e^{(-te-u)\sqrt{\gamma 1}}\left(-1 + e^{u\sqrt{\gamma 1}}\right)\eta 4\left(e^{(2\,te+u)\sqrt{\gamma 1}}\sqrt{\gamma 1}\,\gamma_0 +\right.$$

$$\left.\left.\sqrt{\gamma 1}\left(e^{(te+u)\sqrt{\gamma 1}}\left(-1 + e^{te\sqrt{\gamma 1}}\right)p\sqrt{\gamma 1}\,\delta - e^{(2\,te+u)\sqrt{\gamma 1}}\sqrt{\gamma 1}\,\nu_{c0}\right)\right)\right)\Big/$$

$$\left(\sqrt{2}\sqrt{\left((u + te - \alpha) + \frac{1}{4\,\gamma 1}te\,\eta 3^2\left(-2 + \text{Cosh}\left[2\,(te-u)\sqrt{\gamma 1}\right] + \text{Cosh}\left[\right.\right.\right.}\right.$$

$$\left.\left.\left.\left.2\,(te+u)\sqrt{\gamma 1}\right]\right)\sigma_c^2 + u\,\eta 4^2\,\text{Sinh}\left[u\sqrt{\gamma 1}\right]^2\sigma_f^2\right)\right)\Big/$$

$$\left(1 - \text{CDF}\left[\text{NormalDistribution}\left[0, \text{Sqrt}\left[(u + te - \alpha) + \eta 3^2\right.\right.\right.\right.$$

$$\left(\frac{1}{\gamma 1}\left(\frac{1}{4}\,te\left(-2 + \text{Cosh}\left[2\,(te-u)\sqrt{\gamma 1}\right] +\right.\right.\right.$$

$$\left.\left.\left.\text{Cosh}\left[2\,(te+u)\sqrt{\gamma 1}\right]\right)\right)\right)\sigma_c^2 + \eta 4^2\left(u\,\text{Sinh}\left[u\sqrt{\gamma 1}\right]^2\sigma_f^2\right)\Big]\Big],$$

$$\Bigg(\eta1\ (\text{te} + \text{u} - \alpha) + \eta2\ (\text{te} + \text{u} - \alpha)^2 +$$
$$\eta3\left(\frac{1}{2\,\gamma1}\,e^{-\text{te}\sqrt{\gamma1}}\left(\left(-1 + e^{\text{te}\sqrt{\gamma1}}\right)^2 \gamma_0 + \left(-1 + e^{2\,\text{te}\sqrt{\gamma1}}\right)\sqrt{\gamma1}\ (\text{p}\,\delta - \nu_{c0})\right)\right) +$$
$$\eta4\left(\frac{1}{2\,\gamma1}\,e^{-(\text{te}+\text{u})\sqrt{\gamma1}}\left(-1 + e^{\text{u}\sqrt{\gamma1}}\right)\left(\left(-1 + e^{(2\,\text{te}+\text{u})\sqrt{\gamma1}}\right)\gamma_0 +\right.\right.$$
$$\left.\left.\sqrt{\gamma1}\left(\left(-1 + e^{\text{te}\sqrt{\gamma1}}\right)\left(-1 + e^{(\text{te}+\text{u})\sqrt{\gamma1}}\right)\text{p}\,\delta - \left(1 + e^{(2\,\text{te}+\text{u})\sqrt{\gamma1}}\right)\nu_{c0}\right)\right)\right]\Bigg)\ /.$$

{ $\delta \to 1$, $\eta1 \to -1.5681$, $\eta2 \to 0.2205$`, $\eta3 \to 0.1107$, $\gamma_0 \to 0$,
$\gamma1 \to$ Exp[-21.50524], $\nu_{c0} \to -1.7248$, $\sigma_c \to$ Exp[-27.1608]}

$$
\begin{aligned}
\mathbf{h_f} = \Bigg(& e^{-(A/B)} \\
& \Bigg(-\bigg(\Big(2587.7430878925893` \, e^{-0.00002138929488749056` \, te} \left(-1 + e^{0.00004277858977498112` \, te}\right) (1.7248` + p) - \\
& \quad 1.5681` \, (te + u - \alpha) + 0.2205` \, (te + u - \alpha)^2 + \\
& \quad 23376.179655759614` \, e^{0.00002138929488749056` \, (-te-u)} \left(-1 + e^{0.00002138929488749056` \, u}\right) \\
& \quad \Big(1.7248` \left(1 + e^{0.00002138929488749056` \, (2\,te+u)}\right) + \left(-1 + e^{0.00002138929488749056` \, te}\right) \\
& \quad \left(-1 + e^{0.00002138929488749056` \, (te+u)}\right) p\Big) \, \eta 4\Big) \, \Big(1.7150278872001417`*\text{^}{-17} \; te \\
& \quad (-0.00004277858977498112` \, \mathrm{Sinh}[0.00004277858977498112` \, (te - u)] + \\
& \quad 0.00004277858977498112` \, \mathrm{Sinh}[0.00004277858977498112` \, (te + u)]) + \\
& \quad 0.00004277858977498112` \, u \, \eta 4^2 \, \mathrm{Cosh}[0.00002138929488749056` \, u] \\
& \quad \mathrm{Sinh}[0.00002138929488749056` \, u] \, \sigma_f^2 + \eta 4^2 \, \mathrm{Sinh}[0.00002138929488749056` \, u]^2 \, \sigma_f^2\Big)\bigg) \Big/ \\
& \bigg(2\sqrt{2} \, \Big(te + u - \alpha + 1.7150278872001417`*\text{^}{-17} \; te \, (-2 + \mathrm{Cosh}[0.00004277858977498112` \\
& \quad (te - u)] + \mathrm{Cosh}[0.00004277858977498112` \, (te + u)]) + \\
& \quad u \, \eta 4^2 \, \mathrm{Sinh}[0.00002138929488749056` \, u]^2 \, \sigma_f^2\Big)^{3/2}\bigg) + \\
& \bigg(-1.5681` + 0.441` \, (te + u - \alpha) + 23376.179655759614` \, e^{0.00002138929488749056` \, (-te-u)} \\
& \quad \left(-1 + e^{0.00002138929488749056` \, u}\right) \Big(0.00003689225582194372` \, e^{0.00002138929488749056` \, (2\,te+u)} + \\
& \quad 0.00002138929488749056` \, e^{0.00002138929488749056` \, (te+u)} \left(-1 + e^{0.00002138929488749056` \, te}\right) p\Big) \\
& \quad \eta 4 + 0.5` \, e^{0.00002138929488749056` \, (-te-u)+0.00002138929488749056` \, u} \\
& \quad \Big(1.7248` \left(1 + e^{0.00002138929488749056` \, (2\,te+u)}\right) + \left(-1 + e^{0.00002138929488749056` \, te}\right) \\
& \quad \left(-1 + e^{0.00002138929488749056` \, (te+u)}\right) p\Big) \, \eta 4 - 0.5` \, e^{0.00002138929488749056` \, (-te-u)} \\
& \quad \left(-1 + e^{0.00002138929488749056` \, u}\right) \Big(1.7248` \left(1 + e^{0.00002138929488749056` \, (2\,te+u)}\right) + \\
& \quad \left(-1 + e^{0.00002138929488749056` \, te}\right) \left(-1 + e^{0.00002138929488749056` \, (te+u)}\right) p\Big) \, \eta 4\bigg) \Big/ \\
& \bigg(\sqrt{2} \, \sqrt{\Big(te + u - \alpha + 1.7150278872001417`*\text{^}{-17} \; te \, (-2 + \mathrm{Cosh}[0.00004277858977498112`} \\
& \quad (te - u)] + \mathrm{Cosh}[0.00004277858977498112` \, (te + u)]) + \\
& \quad u \, \eta 4^2 \, \mathrm{Sinh}[0.00002138929488749056` \, u]^2 \, \sigma_f^2\Big)\bigg)\Bigg)\Bigg) \Big/ \\
& \Bigg(\sqrt{\pi} \, \Bigg(1 + \frac{1}{2} \bigg(-1 - \mathrm{Erf}\Big[\Big(2587.7430878925893` \, e^{-0.00002138929488749056` \, te} \\
& \quad \left(-1 + e^{0.00004277858977498112` \, te}\right) (1.7248` + p) - 1.5681` \, (te + u - \alpha) + \\
& \quad 0.2205` \, (te + u - \alpha)^2 + 23376.179655759614` \, e^{0.00002138929488749056` \, (-te-u)} \\
& \quad \left(-1 + e^{0.00002138929488749056` \, u}\right) \Big(1.7248` \left(1 + e^{0.00002138929488749056` \, (2\,te+u)}\right) + \\
& \quad \left(-1 + e^{0.00002138929488749056` \, te}\right) \left(-1 + e^{0.00002138929488749056` \, (te+u)}\right) p\Big) \, \eta 4\Big) \Big/ \\
& \quad \Big(\sqrt{2} \, \sqrt{\Big(te + u - \alpha + 1.7150278872001417`*\text{^}{-17} \; te \, (-2 + \mathrm{Cosh}[} \\
& \quad 0.00004277858977498112` \, (te - u)] + \mathrm{Cosh}[0.00004277858977498112` \\
& \quad (te + u)]) + u \, \eta 4^2 \, \mathrm{Sinh}[0.00002138929488749056` \, u]^2 \, \sigma_f^2\Big)\Big)\Big]\bigg)\Bigg)\Bigg)
\end{aligned}
$$

```
(e^-(A/B) (-((2587.7430878925893` e^-0.00002138929488749056` te (-1 + e^0.00004277858977498112` te) (1.7248` + p) -
      1.5681` (te + u - α) + 0.2205` (te + u - α)^2 +
      23376.179655759614` e^0.00002138929488749056` (-te-u) (-1 + e^0.00002138929488749056` u)
       (1.7248` (1 + e^0.00002138929488749056` (2 te+u)) + (-1 + e^0.00002138929488749056` te)
          (-1 + e^0.00002138929488749056` (te+u)) p) η4) (1.7150278872001417`*^-17 te
       (-0.00004277858977498112` Sinh[0.00004277858977498112` (te - u)] +
         0.00004277858977498112` Sinh[0.00004277858977498112` (te + u)]) +
      0.00004277858977498112` u η4^2 Cosh[0.00002138929488749056` u]
       Sinh[0.00002138929488749056` u] σf^2 + η4^2 Sinh[0.00002138929488749056` u]^2 σf^2)) /
   (2 √2 (te + u - α + 1.7150278872001417`*^-17 te (-2 + Cosh[0.00004277858977498112`
           (te - u)] + Cosh[0.00004277858977498112` (te + u)]) +
      u η4^2 Sinh[0.00002138929488749056` u]^2 σf^2)^3/2) +
   (-1.5681` + 0.441` (te + u - α) + 23376.179655759614` e^0.00002138929488749056` (-te-u)
     (-1 + e^0.00002138929488749056` u) (0.00003689225582194372` e^0.00002138929488749056` (2 te+u) +
      0.00002138929488749056` e^0.00002138929488749056` (te+u) (-1 + e^0.00002138929488749056` te) p) η4 +
    0.5` e^0.00002138929488749056` (-te-u)+0.00002138929488749056` u
     (1.7248` (1 + e^0.00002138929488749056` (2 te+u)) +
       (-1 + e^0.00002138929488749056` te) (-1 + e^0.00002138929488749056` (te+u)) p) η4 -
    0.5` e^0.00002138929488749056` (-te-u) (-1 + e^0.00002138929488749056` u)
     (1.7248` (1 + e^0.00002138929488749056` (2 te+u)) +
       (-1 + e^0.00002138929488749056` te) (-1 + e^0.00002138929488749056` (te+u)) p) η4) /
   (√2 √(te + u - α + 1.7150278872001417`*^-17 te (-2 + Cosh[0.00004277858977498112`
           (te - u)] + Cosh[0.00004277858977498112` (te + u)]) +
      u η4^2 Sinh[0.00002138929488749056` u]^2 σf^2)))) /
 (√π (1 + 1/2 (-1 - Erf[(2587.7430878925893` e^-0.00002138929488749056` te
         (-1 + e^0.00004277858977498112` te) (1.7248` + p) - 1.5681` (te + u - α) +
        0.2205` (te + u - α)^2 + 23376.179655759614` e^0.00002138929488749056` (-te-u)
         (-1 + e^0.00002138929488749056` u) (1.7248` (1 + e^0.00002138929488749056` (2 te+u)) +
           (-1 + e^0.00002138929488749056` te) (-1 + e^0.00002138929488749056` (te+u)) p) η4) /
       (√2 √(te + u - α + 1.7150278872001417`*^-17 te (-2 + Cosh[
                0.00004277858977498112` (te - u)] + Cosh[0.00004277858977498112`
                (te + u)]) + u η4^2 Sinh[0.00002138929488749056` u]^2 σf^2)]))))
```

The survival function for a former-smoker, the exponential of the integral of the negative of the hazard-rate over the period of observation, is given by equation [6.5]]. Note that this equation requires numerical methods to carry out.

[6.5]

```
gf = Exp[Hold[NIntegrate[-hf, {u, 0, ulage}]]]
```

The hazard-rate for former-smokers evaluated at time of death is given by equation [6.6],

[6.6]

$ht_f = h_f$ /. u -> ulage

$$
\begin{aligned}
&\Big(\mathrm{e}^{-(A/B)} \Big(-\Big(\big(2587.7430878925893`\ \mathrm{e}^{-0.00002138929488749056`\,\text{te}} \big(-1+\mathrm{e}^{0.00004277858977498112`\,\text{te}}\big)(1.7248`+p) - \\
&\quad 1.5681`\,(\text{te}+\text{ulage}-\alpha) + 0.2205`\,(\text{te}+\text{ulage}-\alpha)^2 + \\
&\quad 23376.179655759614`\ \mathrm{e}^{0.00002138929488749056`\,(-\text{te}-\text{ulage})} \big(-1+\mathrm{e}^{0.00002138929488749056`\,\text{ulage}}\big) \\
&\quad \big(1.7248`\ \big(1+\mathrm{e}^{0.00002138929488749056`\,(2\,\text{te}+\text{ulage})}\big) + \big(-1+\mathrm{e}^{0.00002138929488749056`\,\text{te}}\big) \\
&\quad \big(-1+\mathrm{e}^{0.00002138929488749056`\,(\text{te}+\text{ulage})}\big)\,p\big)\,\eta4\big)\ \big(1.7150278872001417`{*}\hat{}{-17}\ \text{te} \\
&\quad (-0.00004277858977498112`\ \text{Sinh}[0.00004277858977498112`\ (\text{te}-\text{ulage})] + \\
&\quad 0.00004277858977498112`\ \text{Sinh}[0.00004277858977498112`\ (\text{te}+\text{ulage})]) + \\
&\quad 0.00004277858977498112`\ \text{ulage}\ \eta4^2\ \text{Cosh}[0.00002138929488749056`\ \text{ulage}] \\
&\quad \text{Sinh}[0.00002138929488749056`\ \text{ulage}]\ \sigma_f^2 + \\
&\quad \eta4^2\ \text{Sinh}[0.00002138929488749056`\ \text{ulage}]^2\ \sigma_f^2\big)\Big) \Big/ \\
&\Big(2\sqrt{2}\ \big(\text{te}+\text{ulage}-\alpha+1.7150278872001417`{*}\hat{}{-17}\ \text{te}\ (-2+\text{Cosh}[0.00004277858977498112` \\
&\quad (\text{te}-\text{ulage})] + \text{Cosh}[0.00004277858977498112`\ (\text{te}+\text{ulage})]) + \\
&\quad \text{ulage}\ \eta4^2\ \text{Sinh}[0.00002138929488749056`\ \text{ulage}]^2\ \sigma_f^2\big)^{3/2}\Big) + \\
&\Big(-1.5681` + 0.441`\,(\text{te}+\text{ulage}-\alpha) + 23376.179655759614`\ \mathrm{e}^{0.00002138929488749056`\,(-\text{te}-\text{ulage})} \\
&\quad \big(-1+\mathrm{e}^{0.00002138929488749056`\,\text{ulage}}\big)\ \big(0.00003689225582194372`\ \mathrm{e}^{0.00002138929488749056`\,(2\,\text{te}+\text{ulage})} + \\
&\quad 0.00002138929488749056`\ \mathrm{e}^{0.00002138929488749056`\,(\text{te}+\text{ulage})} \big(-1+\mathrm{e}^{0.00002138929488749056`\,\text{te}}\big)\,p\big) \\
&\quad \eta4 + 0.5`\ \mathrm{e}^{0.00002138929488749056`\,(-\text{te}-\text{ulage})+0.00002138929488749056`\,\text{ulage}} \\
&\quad \big(1.7248`\ \big(1+\mathrm{e}^{0.00002138929488749056`\,(2\,\text{te}+\text{ulage})}\big) + \\
&\quad \big(-1+\mathrm{e}^{0.00002138929488749056`\,\text{te}}\big)\ \big(-1+\mathrm{e}^{0.00002138929488749056`\,(\text{te}+\text{ulage})}\big)\,p\big)\,\eta4 - \\
&\quad 0.5`\ \mathrm{e}^{0.00002138929488749056`\,(-\text{te}-\text{ulage})}\ \big(-1+\mathrm{e}^{0.00002138929488749056`\,\text{ulage}}\big) \\
&\quad \big(1.7248`\ \big(1+\mathrm{e}^{0.00002138929488749056`\,(2\,\text{te}+\text{ulage})}\big) + \\
&\quad \big(-1+\mathrm{e}^{0.00002138929488749056`\,\text{te}}\big)\ \big(-1+\mathrm{e}^{0.00002138929488749056`\,(\text{te}+\text{ulage})}\big)\,p\big)\,\eta4\Big) \Big/ \\
&\Big(\sqrt{2}\ \sqrt{}\big(\text{te}+\text{ulage}-\alpha+1.7150278872001417`{*}\hat{}{-17}\ \text{te}\ (-2+\text{Cosh}[0.00004277858977498112` \\
&\quad (\text{te}-\text{ulage})] + \text{Cosh}[0.00004277858977498112`\ (\text{te}+\text{ulage})]) + \\
&\quad \text{ulage}\ \eta4^2\ \text{Sinh}[0.00002138929488749056`\ \text{ulage}]^2\ \sigma_f^2\big)\Big)\Big)\Big) \Big/ \\
&\Big(\sqrt{\pi}\ \Big(1+\frac{1}{2}\ \Big(-1-\text{Erf}\Big[\big(2587.7430878925893`\ \mathrm{e}^{-0.00002138929488749056`\,\text{te}} \big(-1+\mathrm{e}^{0.00004277858977498112`\,\text{te}}\big) \\
&\quad (1.7248`+p) - 1.5681`\,(\text{te}+\text{ulage}-\alpha) + 0.2205`\,(\text{te}+\text{ulage}-\alpha)^2 + \\
&\quad 23376.179655759614`\ \mathrm{e}^{0.00002138929488749056`\,(-\text{te}-\text{ulage})} \big(-1+\mathrm{e}^{0.00002138929488749056`\,\text{ulage}}\big) \\
&\quad \big(1.7248`\ \big(1+\mathrm{e}^{0.00002138929488749056`\,(2\,\text{te}+\text{ulage})}\big) + \big(-1+\mathrm{e}^{0.00002138929488749056`\,\text{te}}\big) \\
&\quad \big(-1+\mathrm{e}^{0.00002138929488749056`\,(\text{te}+\text{ulage})}\big)\,p\big)\,\eta4\big) \Big/ \\
&\quad \big(\sqrt{2}\ \sqrt{}\big(\text{te}+\text{ulage}-\alpha+1.7150278872001417`{*}\hat{}{-17}\ \text{te}\ (-2+\text{Cosh}[\\
&\quad 0.00004277858977498112`\ (\text{te}-\text{ulage})] + \text{Cosh}[0.00004277858977498112` \\
&\quad (\text{te}+\text{ulage})]) + \text{ulage}\ \eta4^2\ \text{Sinh}[0.00002138929488749056`\ \text{ulage}]^2\ \sigma_f^2\big)\big)\Big]\Big)\Big)\Big)
\end{aligned}
$$

The probability density function of the random variable "life span" for former-smokers is the product of the hazard-rate evaluated at time of death and the survival function evaluated at time of death, equation [6.7],

[6.7]

$$ft_f = ht_f \; g_f$$

which can be obtained by substitution.

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