

Supplement 4: sensitivity analysis for Pätši et al.

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Assumption checks & sensitivity analyses

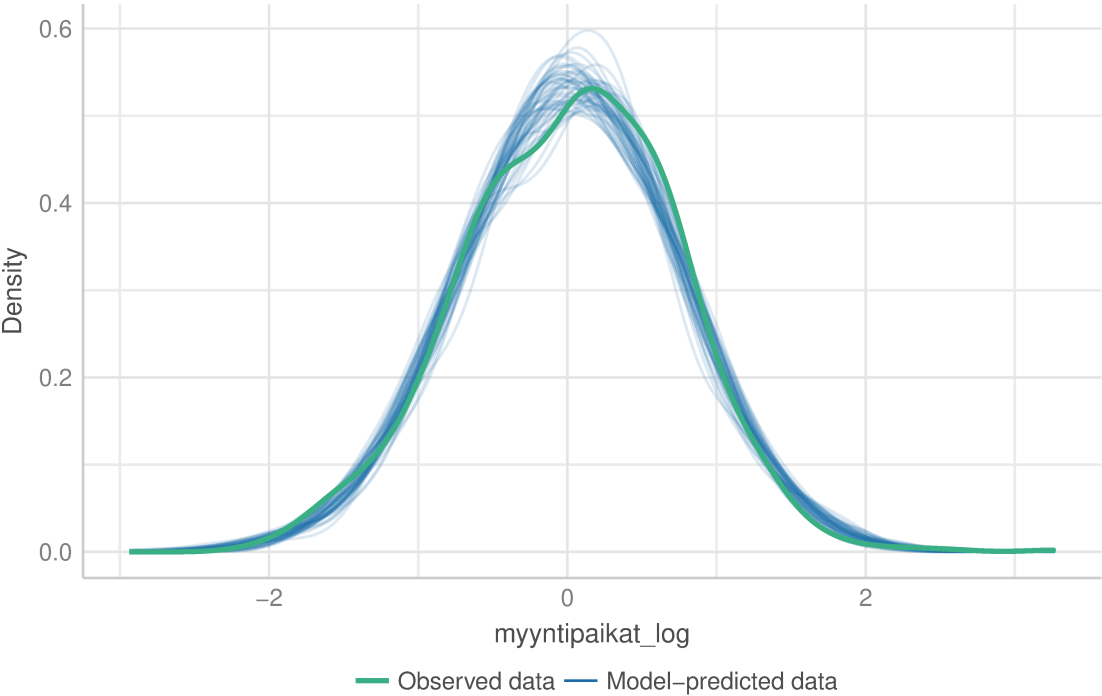
Posterior predictive checks are within expected range.

Linearity, homogeneity of variance and leverage identify a potential problem data point. Linearity also shows a minor uptick at the low end. To see the effect of the influential data point, Sensitivity analysis #1: removes outliers.

Median income and % of low income are potentially multicollinear. To see the effect of multicollinearity, Sensitivity analysis #2: use only one income variable.

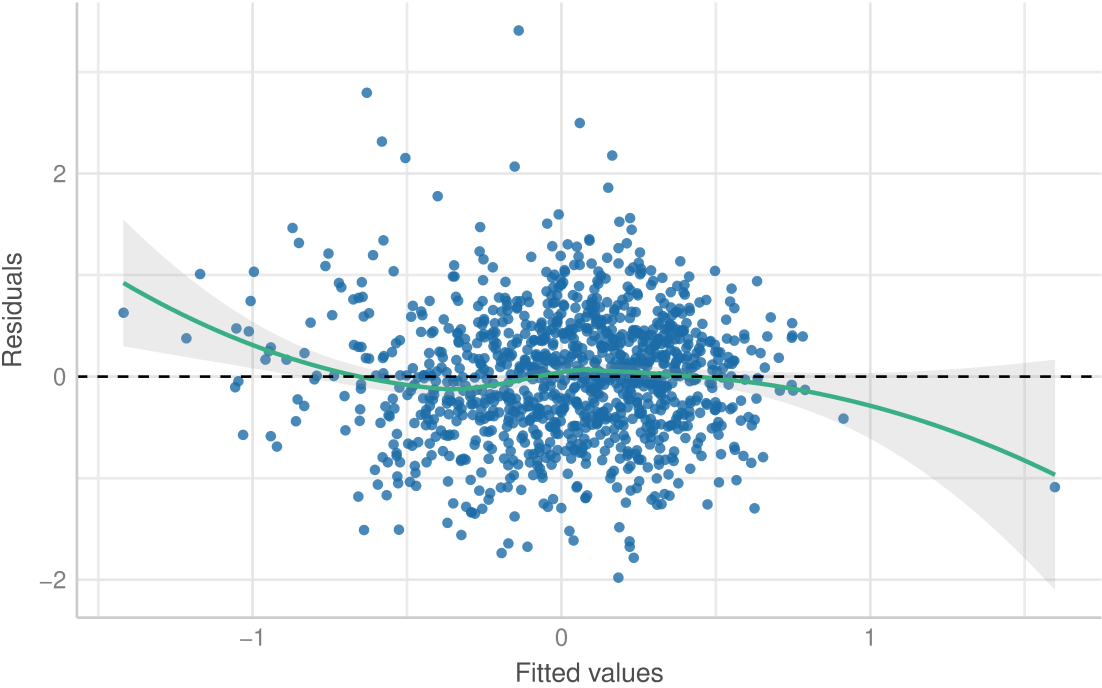
## \$PP\_CHECK

Posterior Predictive Check  
Model-predicted lines should resemble observed data line



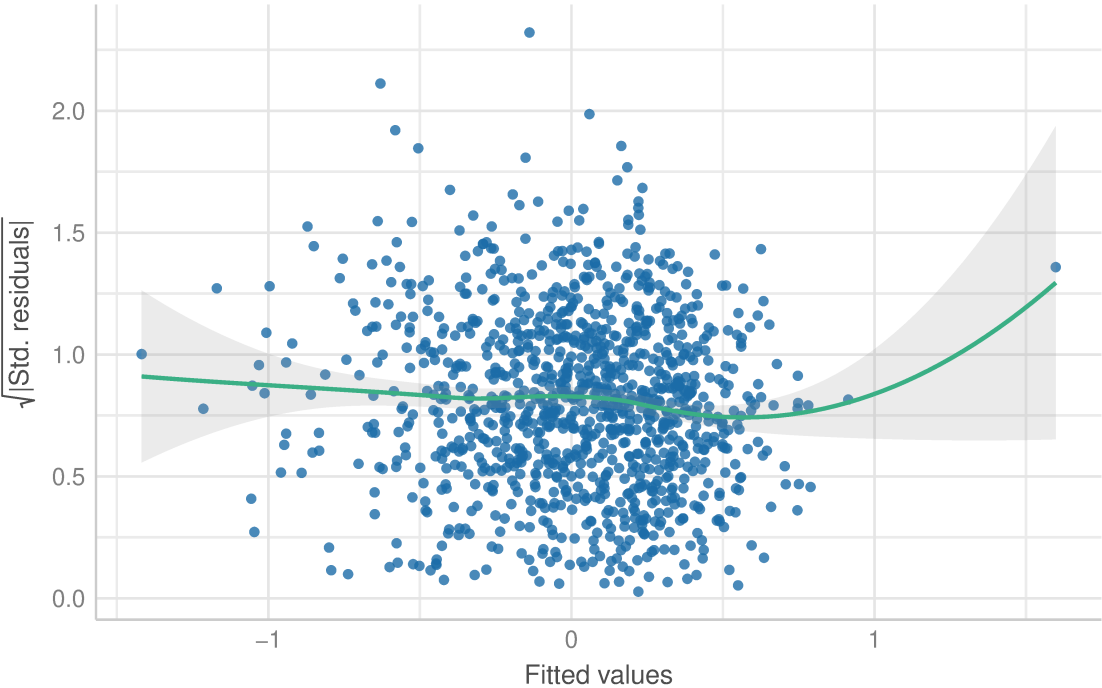
##  
## \$NCV

Linearity  
Reference line should be flat and horizontal



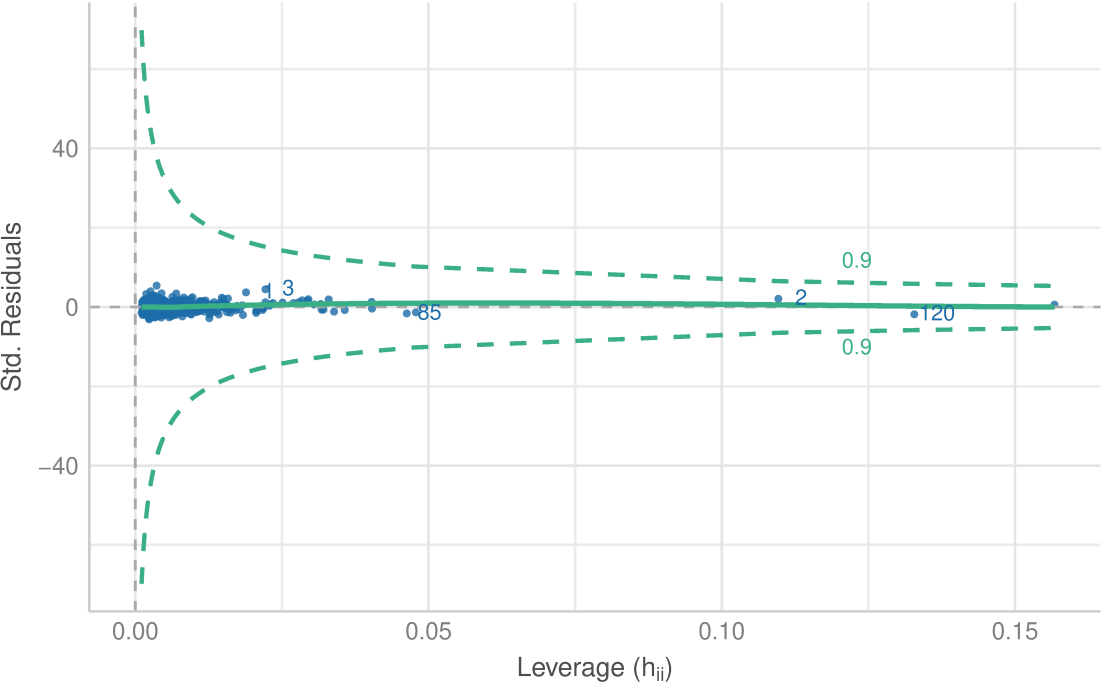
##  
## \$HOMOGENEITY

Homogeneity of Variance  
Reference line should be flat and horizontal



##  
## \$OUTLIERS

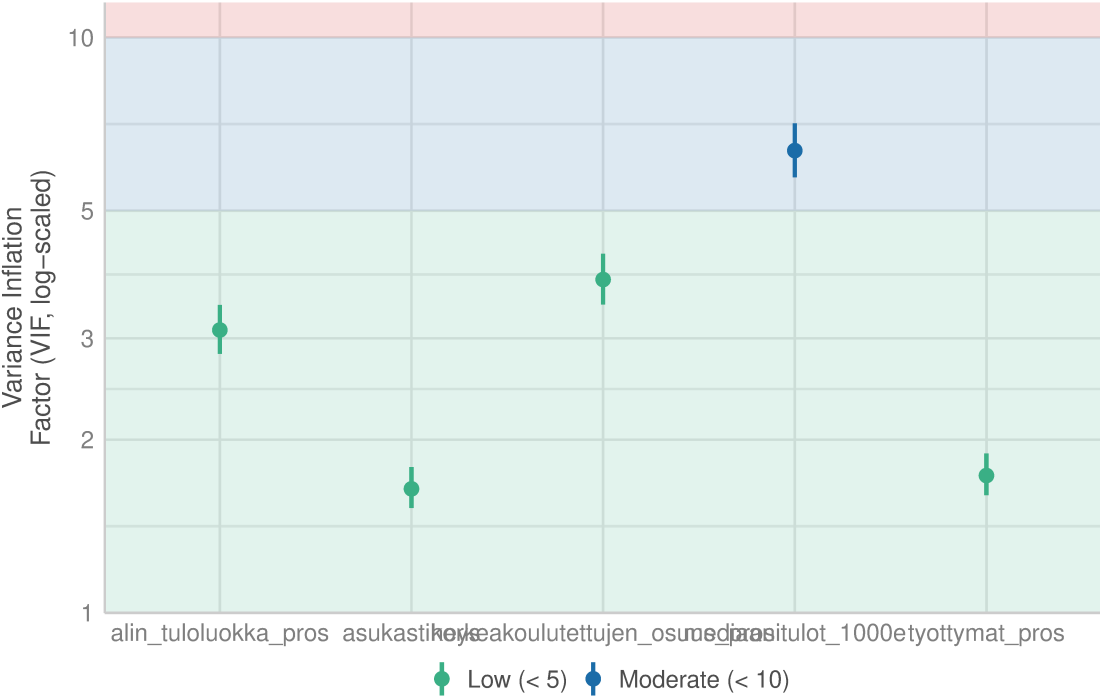
Influential Observations  
Points should be inside the contour lines



##  
## \$VIF

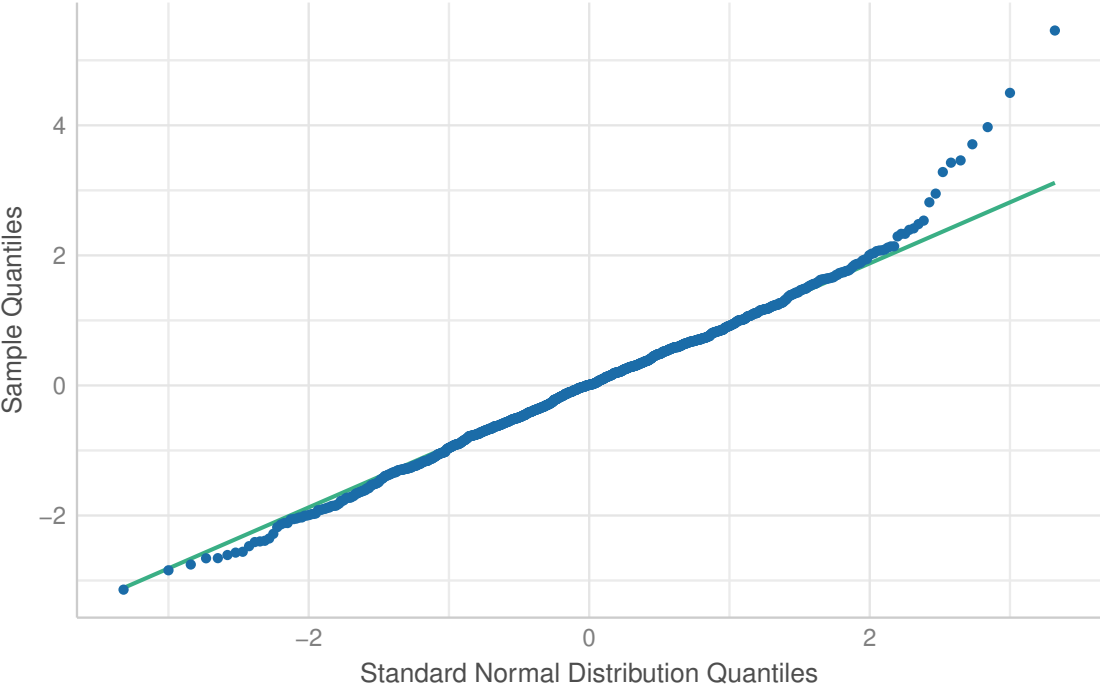
Collinearity

High collinearity (VIF) may inflate parameter uncertainty



##  
## \$QQ

Normality of Residuals  
Dots should fall along the line



Sensitivity #1: Remove high leverage points

The influential data points are those with an unusually high density of retail locations. Removing all locations with  $\log(\text{density}) > 2.5$  or density > 12.18 to see if this changes results.

Coefficient changes are small and removal of outliers/high leverage points does not affect conclusions.

Table 1: Multivariate associations for presence of retailer and retailer density by SES, density outliers removed

Variable	Logistic regression for presence of retailer			Linear regression for log of retailer density		
	Estimate	SE	P value	Estimate	SE	P value
Median income, 1000 euro	0.543	0.060	<0.001	0.957	0.014	0.002
% in lowest income category	0.885	0.028	<0.001	1.029	0.007	<0.001
% unemployed	1.064	0.026	0.018	0.977	0.006	<0.001
% with higher education	1.139	0.017	<0.001	0.983	0.003	<0.001
Population density, 1000s per square km	2.911	0.250	<0.001	1.033	0.014	0.020

Estimates are exponentiated and can be interpreted as multiplicative increases in probability and density, given 1 unit change in predictor

Sensitivity #2: remove potentially collinear variables

Conventional guidelines for VIF are that if the largest VIF is >10, there is evidence of consequential multicollinearity, and if >5 there is cause for concern. However, it is important to note that as O’Brien (2007) shows VIF values as high as 40 do not necessarily mean the regression is wrong or call for the elimination of one or more independent variables, and best practice is to expli

Here are the VIF values. For both models, they are lower than 10, but for income they are higher than 5, meaning there is some reason for concern. Sensitivity analysis #2 above explores what happens if we remove the highest VIF variable. For the linear model, removing one of the income variables does not change the results in a way that would change the conclusions. For the logistic model, removing median income changes the direction of the lowest income category from negative and significant to positive and non-significant. This has little substantial effect on the conclusions.

	x
mediaanitulot_1000e	6.358342
alin_tuloluokka_pros	3.099495
tyottomat_pros	1.731645
korkeakoulutettujen_osuus_pros	3.796562
asukastiheys	1.642141

	x
mediaanitulot_1000e	6.869882
alin_tuloluokka_pros	2.922583
tyottomat_pros	1.702085
korkeakoulutettujen_osuus_pros	4.483845
asukastiheys	1.475021

	Logistic regression for presence of retailer			Linear regression for log of retailer density		
Variable	Estimate	SE	P value	Estimate	SE	P value
% in lowest income category	1.047	0.022	0.038	1.043	0.005	<0.001
% unemployed	1.178	0.025	<0.001	0.981	0.006	<0.001
% with higher education	1.023	0.012	0.060	0.976	0.003	<0.001
Population density, 1000s per square km	3.092	0.229	<0.001	1.041	0.014	0.005

\caption{Multivariate associations for presence of retailer and retailer density by SES, % in lowest income  
dropped}

	Logistic regression for presence of retailer			Linear regression for log of retailer density		
Variable	Estimate	SE	P value	Estimate	SE	P value
Median income, 1000€	0.620	0.050	<0.001	0.917	0.010	<0.001
% unemployed	1.056	0.026	0.039	0.977	0.006	<0.001
% with higher education	1.137	0.017	<0.001	0.986	0.003	<0.001
Population density, 1000s per square km	2.843	0.245	<0.001	1.033	0.014	0.026

\end{table}

An alternative to the log-normal model would be a general linear model with a log-link function. In this analysis, the main results remain substantially the same. The effects of % in lowest income category and

population density are brought closer to 1 or no effect and become non-significant, but the others remain similar and significant.

Table 3: Multivariate associations for presence of retailer and retailer density by SES, general linear model with log-link

Variable	Logistic regression for presence of retailer			General linear model with log-link		
	Estimate	SE	P value	Estimate	SE	P value
Median income, 1000s €	0.542	0.060	<0.001	0.933	0.026	0.007
% in lowest income category	0.884	0.028	<0.001	1.004	0.010	0.696
% unemployed	1.063	0.026	0.020	0.979	0.008	0.007
% with higher education	1.139	0.017	<0.001	0.987	0.005	0.018
Population density, 1000s per square km	2.903	0.250	<0.001	0.993	0.030	0.809

Estimates are exponentiated and can be interpreted as multiplicative increases in probability and density, given 1 unit change in predictor

## Sensitivity check #4: Zero-inflated Gamma-regression

Another alternative model would be zero-inflated Gamma-regression, a model that models both for the absence of retailer (note: this is the opposite of the logistic model, which models for presence of retailer) and the density at the same time, in one model.

Results are the same as the models in the main paper (to compare the zero-inflated coefficients to the logistic regression coefficients, take the reciprocal 1/coefficient).

Table 4: Multivariate associations for presence of retailer and retailer density by SES, zero-inflated gamma

component	term	estimate	std.error	statistic	p.value
cond	Intercept	2.781	0.389	2.627	0.009
cond	Median income, 1000s €	0.964	0.014	-2.599	0.009
cond	% in lowest income category	1.033	0.007	4.699	0.000
cond	% unemployed	0.985	0.004	-4.111	0.000
cond	% with higher education	0.970	0.006	-4.868	0.000
cond	Population density, 1000s per square km	1.035	0.014	2.495	0.013
zi	Intercept	0.000	1.670	-8.631	0.000
zi	Median income, 1000s €	1.845	0.060	10.244	0.000
zi	% in lowest income category	1.132	0.028	4.423	0.000
zi	% unemployed	0.878	0.017	-7.561	0.000
zi	% with higher education	0.941	0.026	-2.335	0.020
zi	Population density, 1000s per square km	0.345	0.250	-4.258	0.000

<sup>a</sup> cond = density, zi = Probability for NO sales

## Sensitivity check #5: three income tertiles

To see if including % of high income along with % of low income, essentially making middle income a comparison group, that was tested. However, the high correlations between high income % and low income % make variance inflation issues worse and add instability to estimates.

“Median income, 1000€” = mediaanitulot\_1000e % in lowest income category = alin\_tuloluokka\_pros “% unemployed” = tyottomat\_pros “% with higher education” = korkeakoulutettujen\_osuus\_pros  
 “Population density, 1000s per square km” = asukastiheys



term	estimate	std.error	statistic	p.value
(Intercept)	-1.1261242	0.0838618	-13.42833	0
alin_tuloluokka_pros	0.0574723	0.0040850	14.06899	0

term	estimate	std.error	statistic	p.value
(Intercept)	-0.2473787	0.1499824	-1.649385	0.0993523
alin_tuloluokka_pros	0.0332116	0.0052918	6.276094	0.0000000
high_income	-0.0208686	0.0029807	-7.001174	0.0000000

term	estimate	std.error	statistic	p.value
(Intercept)	-0.1099712	0.2836364	-0.3877188	0.6982241
alin_tuloluokka_pros	0.0689464	0.0147030	4.6892784	0.0000027

term	estimate	std.error	statistic	p.value
(Intercept)	1.4806240	0.5124606	2.889245	0.0038617
alin_tuloluokka_pros	0.0225520	0.0188901	1.193853	0.2325356
high_income	-0.0341776	0.0092772	-3.684040	0.0002296

term	estimate	std.error	statistic	p.value
(Intercept)	2.2323313	0.7157267	3.1189716	0.0018618
mediaanitulos_1000e	-0.0694592	0.0255623	-2.7172494	0.0066856
alin_tuloluokka_pros	0.0038414	0.0098272	0.3908955	0.6959498
tyottomat_pros	-0.0213044	0.0078538	-2.7126108	0.0067792
korkeakoulutettujen_osuus_pros	-0.0129035	0.0054688	-2.3594901	0.0184732
asukastiheys	-0.0072266	0.0299004	-0.2416895	0.8090656

term	estimate	std.error	statistic	p.value
(Intercept)	3.4471968	0.6592186	5.229217	0.0000002
mediaanitulos_1000e	-0.1660028	0.0301337	-5.508874	0.0000000
alin_tuloluokka_pros	0.0093624	0.0078749	1.188888	0.2347394
high_income	0.0451081	0.0098383	4.584962	0.0000051
tyottomat_pros	-0.0232745	0.0059939	-3.883050	0.0001093
korkeakoulutettujen_osuus_pros	-0.0231226	0.0035690	-6.478758	0.0000000
asukastiheys	0.0322759	0.0142543	2.264291	0.0237495

	X
mediaanitilot_1000e	27.443896
alin_tuloluokka_pros	4.159069
high_income	20.459588
tyottomat_pros	1.737190
korkeakoulutettujen_osuus_pros	4.244679
asukastiheys	1.642769

term	estimate	std.error	statistic	p.value
(Intercept)	14.4118729	1.6695759	8.632056	0.0000000
mediaanitilot_1000e	-0.6124198	0.0597751	-10.245405	0.0000000
alin_tuloluokka_pros	-0.1236361	0.0279513	-4.423275	0.0000097
tyottomat_pros	0.0612427	0.0262303	2.334810	0.0195533
korkeakoulutettujen_osuus_pros	0.1298752	0.0171756	7.561591	0.0000000
asukastiheys	1.0656341	0.2502250	4.258703	0.0000206

term	estimate	std.error	statistic	p.value
(Intercept)	15.8387385	2.4103911	6.5710244	0.0000000
mediaanitilot_1000e	-0.6846577	0.1062445	-6.4441678	0.0000000
alin_tuloluokka_pros	-0.1352138	0.0313498	-4.3130694	0.0000161
high_income	0.0271425	0.0328255	0.8268727	0.4083092
tyottomat_pros	0.0622116	0.0262678	2.3683622	0.0178670
korkeakoulutettujen_osuus_pros	0.1259852	0.0177514	7.0971925	0.0000000
asukastiheys	1.0639583	0.2500560	4.2548798	0.0000209

	X
mediaanitulot_1000e	21.591972
alin_tuloluokka_pros	3.665652
high_income	14.850653
tyottomat_pros	1.705356
korkeakoulutettujen_osuus_pros	4.774928
asukastiheys	1.470565